Welcome to a few sample chapters of Option Pricing— Black-Scholes Made Easy

I wrote the software and book *Option-Pricing: Black-Scholes Made Easy in the late 1990s*. John Wiley & Sons published the work in 2001.

For free, you can download the .exe simulator that accompanied the 2001 edition from Wiley. The 2001-version of the simulator is 480 x 640— which is tiny on today's screens. It works only on a PC or on a Mac that has Windows-emulation software.

The Wiley site refers visitors to www.optionanimation.com as my website. I no longer own that domain. Someone else owns it and is illegally plagarizing my work there.

I subsequently developed a larger and improved web version of the simulator, but it ran as an Adobe Shockwave file. As far as I know, browers no longer support Shockwave.

If you like the sample chapters,
you can order the whole book from Amazon.com:
http://www.amazon.com/gp/product/0471436410 .

If you want to see a similar educational approach applied to binomial options pricing theory, download for free my *How To Value Stock Options In Divorce Proceedings Seminar Notes* http://www.jerrymarlow/pdfs/Marlow_How_To_Value_Stock_Options_In_Divorce_Proceedings_Seminar_Notes.pdf .

I can help you achieve amazing things.

My career is devoted to empowering people and making difficult topics easy for just about anyone to understand. If you hav a target audience that you would like to enlighten, energize and empower, be in touch.

I can write speeches and create presentations for you that get your audience excited about your ideas, about your mission and about you.

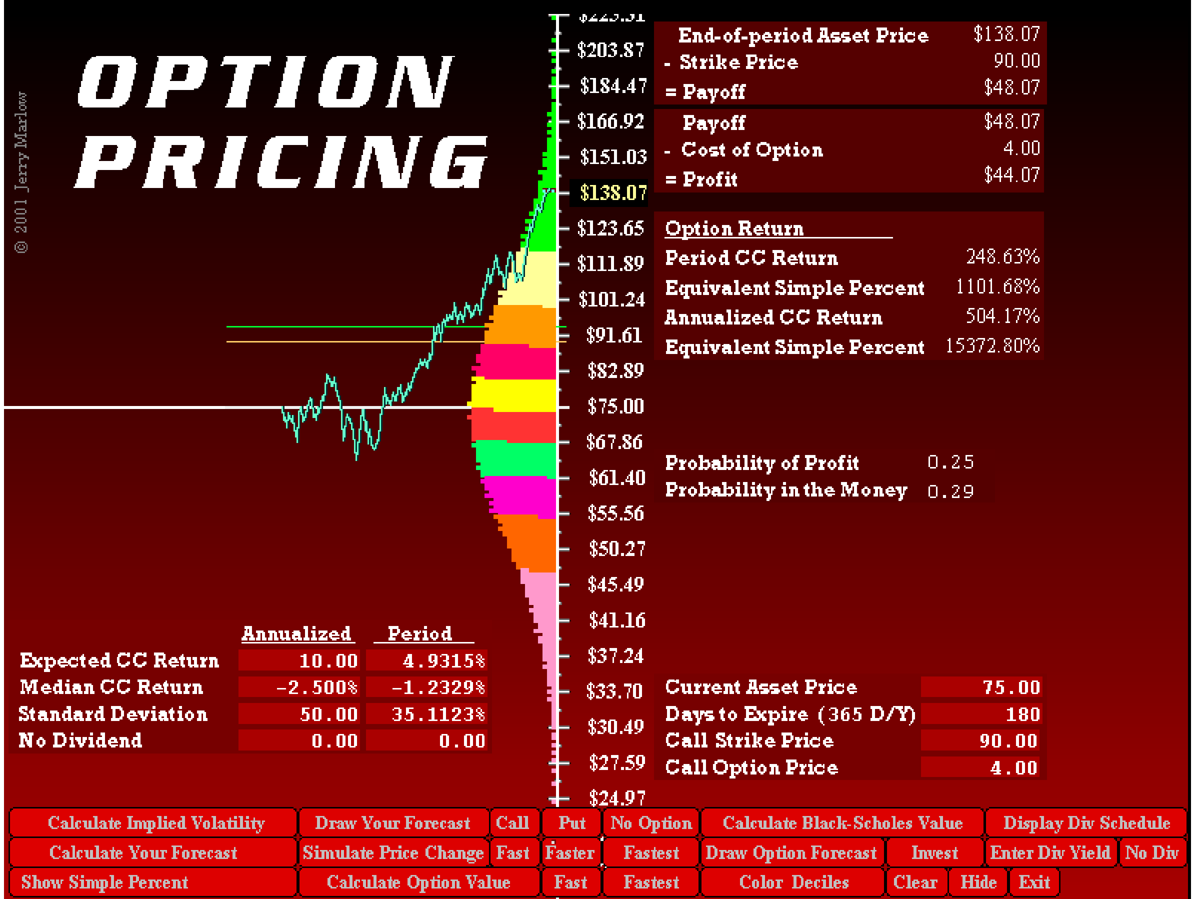
Chances are, I can do for you
what I've done for many
of my other clients—
get you a standing ovation.

If you think I might be able to help you achieve your goals, visit my website at www.jerrymarlow.com,
call me at +1 (917) 817-8659 or email jerrymarlow@jerrymarlow.com.

Let's see if we can get you a standing ovation.

Jerry Marlow
October 2018
New York, NY

OPTION PRICING



Black-Scholes Made Easy

A Visual Way to Understand Stock Options, Option Prices, and Stockmarket Volatility

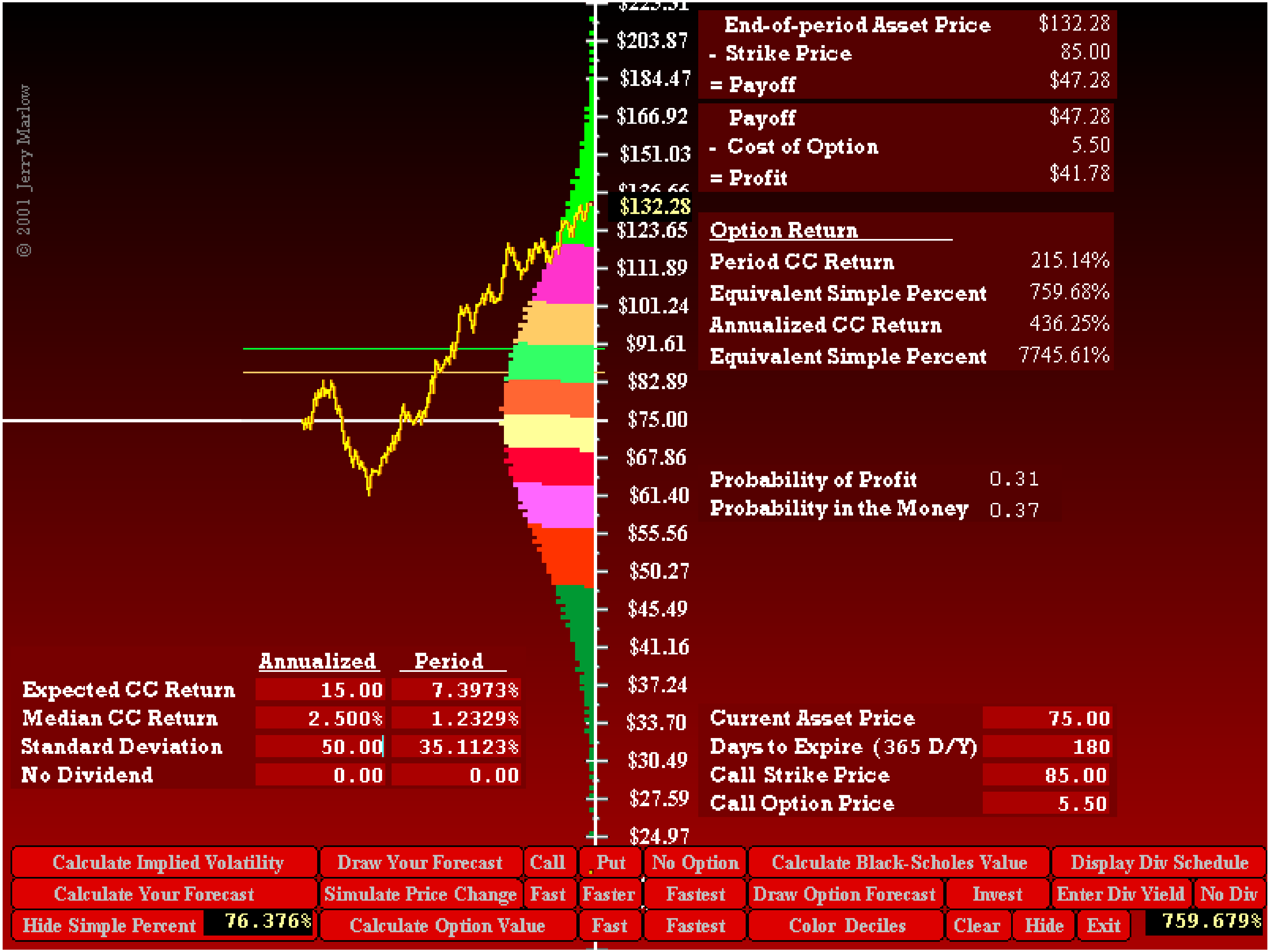
Includes Interactive Animated Tutorial on CD-ROM

Jerry Marlow

Jerry Marlow

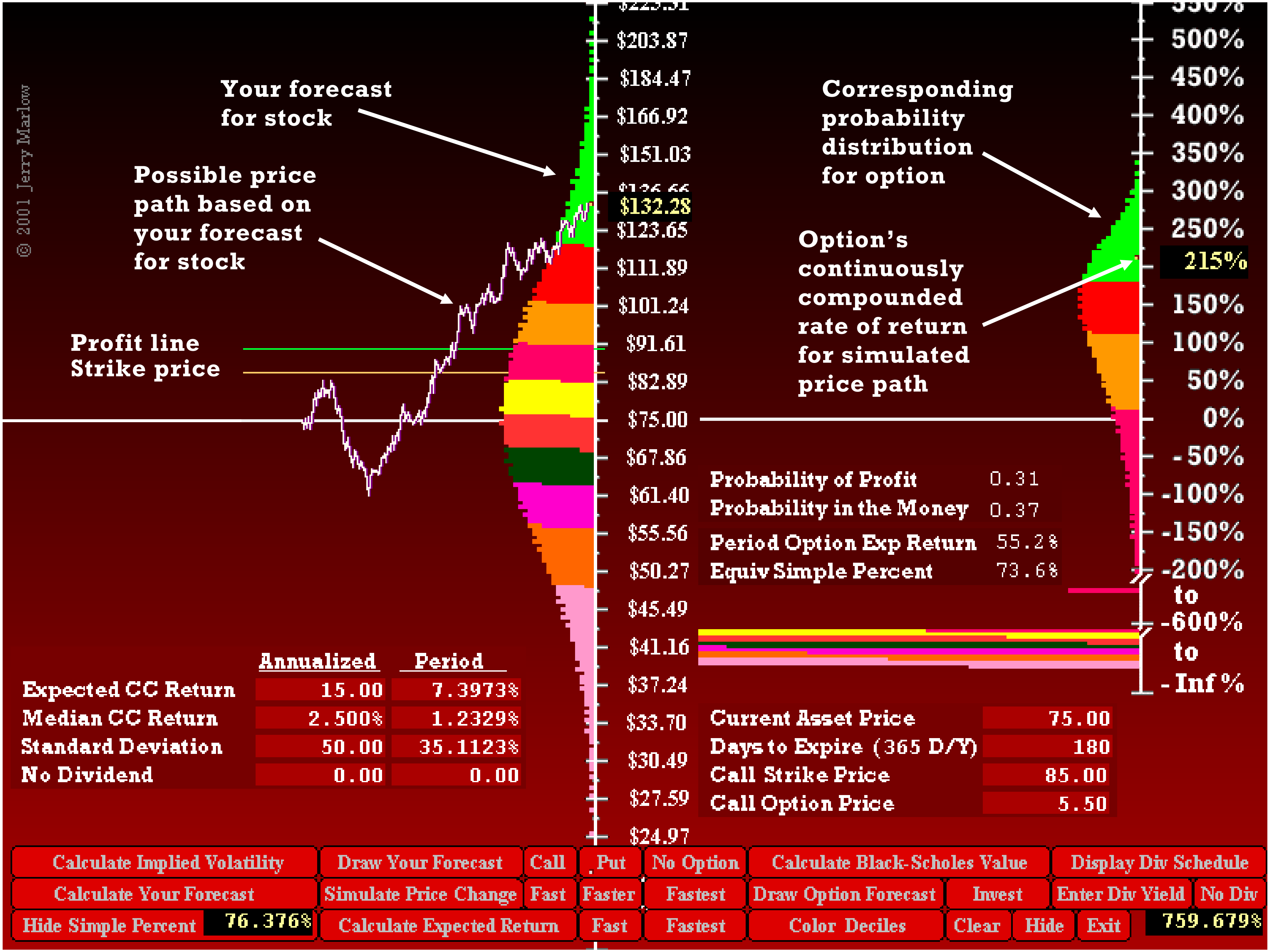
Option Pricing: Black-Scholes Made Easy

A Visual Way to Understand Stock Options, Option Prices, and Stock-Market Volatility


© 2001 Jerry Marlow
End-of-period Asset Price $132.28
- Strike Price 85.00
= Payoff $47.28
Payoff $47.28
- Cost of Option 5.50
= Profit $41.78
Option Return
Period CC Return 215.14%
Equivalent Simple Percent 759.68%
Annualized CC Return 436.25%
Equivalent Simple Percent 7745.61%
Probability of Profit 0.31
Probability in the Money 0.37
$203.87
$184.47
$166.92
$151.03
$132.28
$123.65
$111.89
$101.24
$91.61
$82.89
$75.00
$67.86
$61.40
$55.56
$50.27
$45.49
$41.16
$37.24
$33.70
$30.49
$27.59
$24.97
Annualized Period
Expected CC Return 15.00 7.3973%
Median CC Return 2.500% 1.2329%
Standard Deviation 50.00 35.1123%
No Dividend 0.00 0.00
Current Asset Price 75.00
Days to Expire (365 D/Y) 180
Call Strike Price 85.00
Call Option Price 5.50
Calculate Implied Volatility
Draw Your Forecast
Call
Put
No Option
Calculate Black-Scholes Value
Display Div Schedule
Calculate Your Forecast
Simulate Price Change
Fast
Faster
Fastest
Draw Option Forecast
Invest
Enter Div Yield
No Div
Hide Simple Percent
76.376%
Calculate Option Value
Fast
Fastest
Color Deciles
Clear
Hide
Exit
759.679%

The great thing about investing in options is that
you can make money when the market's going up,
when it's going down, and—if you have the courage
to sell options—when the market isn't going anywhere at all.

Hedge Fund Manager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Your forecast for stock
Possible price path based on your forecast for stock
Profit line
Strike price
Corresponding probability distribution for option
Option's continuously compounded rate of return for simulated price path
$203.87
$184.47
$166.92
$151.03
$132.28
$123.65
$111.89
$101.24
$91.61
$82.89
$75.00
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$61.40
$55.56
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$45.49
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$30.49
$27.59
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500%
450%
400%
350%
300%
250%
215%
150%
100%
50%
0%
-50%
-100%
-150%
-200%
to
-600%
to
-Inf%
Probability of Profit 0.31
Probability in the Money 0.37
Period Option Exp Return 55.2%
Equiv Simple Percent 73.6%
Annualized
Period
Expected CC Return 15.00 7.3973%
Median CC Return 2.500% 1.2329%
Standard Deviation 50.00 35.1123%
No Dividend 0.00 0.00
Current Asset Price 75.00
Days to Expire (365 D/Y) 180
Call Strike Price 85.00
Call Option Price 5.50
Calculate Implied Volatility
Draw Your Forecast
Call
Put
No Option
Calculate Black-Scholes Value
Display Div Schedule
Calculate Your Forecast
Simulate Price Change
Fast
Faster
Fastest
Draw Option Forecast
Invest
Enter Div Yield
No Div
Hide Simple Percent
76.376%
Calculate Expected Return
Fast
Fastest
Color Deciles
Clear
Hide
Exit
759.679%

"The true logic of this world is in the calculus of probabilities."

James Clerk Maxwell

A few years ago, a senior strategist at one of the world's leading investment firms told me something quite profound.

"Many people," he said, "gain some understanding of the stock market. Then they apply that way of thinking to the options market. Usually these people do not fare very well.

"A much smaller group of people gain a sophisticated understanding of options-pricing theory. They then apply this way of thinking to the options market *and to the stock market*. These people have a superior understanding of both markets. They tend to fare extremely well."

In 1997, the Nobel Prize in Economics was awarded for the work that led to Black-Scholes Options-Pricing Theory. Black-Scholes has become the fundamental way of understanding the relationships among options prices, stock forecasts and expected stock-market volatility.

Black-Scholes Options-Pricing Theory is based in the mathematics of probability distributions. Unfortunately, because of the way Black-Scholes usually is presented, many people find the theory's advanced mathematics daunting.

Option Pricing: Black-Scholes Made Easy makes this sophisticated way of thinking accessible to people who do not have the backgrounds necessary to do Nobel-Prize-winning mathematics.

Black-Scholes Made Easy shows you animations and simulations that you can understand easily and intuitively. The mathematics of Black-Scholes and probability distributions is behind the screen driving the animations and simulations.

Animations and simulations review the basics of how options work. They show you the relationships among option prices, stock-market volatility, and financial forecasts.

Animations show you that every financial forecast is a probability distribution and what that means. Simulations give you a clear understanding of what investment professionals mean when they talk about "expected return." The expression may not mean what you think!

Once you have worked through the book and animations, you will understand how market-equilibrium forecasts are embedded in option prices. Using the animations, you will be able to extract from option prices the market-equilibrium forecasts of stocks you are interested in. You will see how, if you disagree with any of those forecasts, you can use options to leverage your expected return.

Based on your forecast, the animation calculates the expected return, probability of profit, and probability of being in the money for options you hold to maturity. You can simulate potential payoffs of investments you have in mind.

Black-Scholes Options-Pricing Theory revealed that investing in options is a probability game. *Black-Scholes Made Easy* shows you your odds.

Jerry Marlow

New York City

July 2001

Jerry Marlow is a freelance financial writer and marketing consultant. For investment firms, he creates marketing and educational presentations that bridge the gap between how sophisticated financial managers think about investing and how the firms' clients think about investing. He holds an MBA in Marketing from New York University where he also did post-MBA work in Finance.

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Acknowledgements

I have had the good fortune to have as friends and clients some very smart and savvy people. They have taught me a great deal about stock options, investment strategies, investor psychology, and about communicating financial theory to investors. For inspiring and enlightening me, I especially thank Peter Lengyel, Josh Weinreich, Rich Marin, and Al Bellino.

Black-Scholes Made Easy is not only a book. It is also a computer animation that contains roughly 10,000 lines of computer code. It is written in Macromedia Director's™ Lingo™. For his lucid and well-organized books on Lingo, I thank Gary Rosenzweig.

For testing the software from a naïve investor's point of view and suggesting ways to make workingn with the animation a smooth interaction, I thank Lianna Sugarman.

For cajoling and encouraging me to address content issues I would've found it easier not to, I thank my gracious and diplomatic editor at John Wiley & Sons Claudio Capanzano.

For alerting me to points on which academicians are apt to attack any work on Black-Scholes, I thank New York University Professor of Mathematics Jonathan Goodman.

For adapting John Wiley & Sons's scholarly style guidelines to this venture in new media and to a writing style geared to oral and informal presentation, I thank Robin Factor.

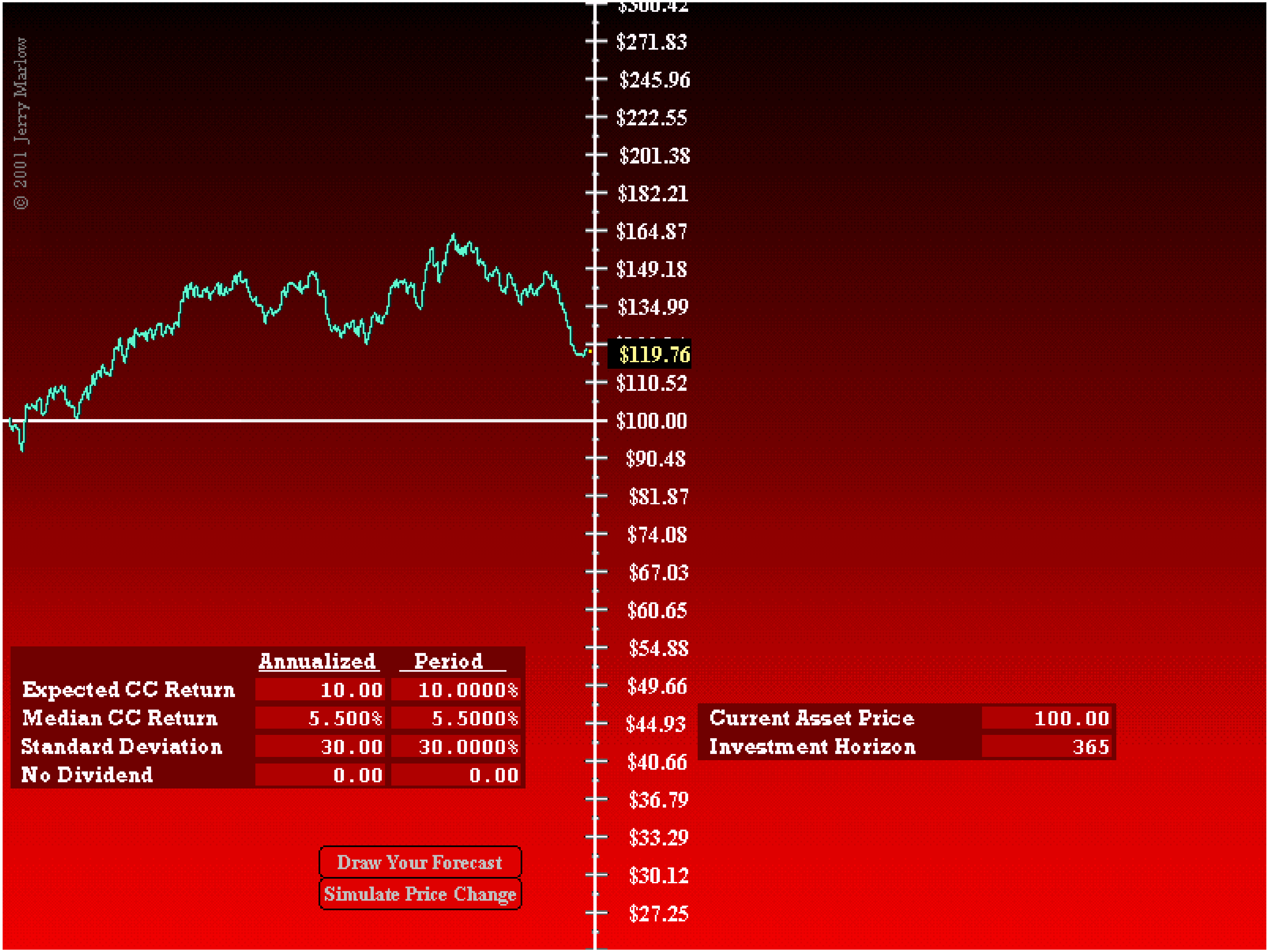
For developing Black-Scholes Options Pricing Theory in the first place, I thank Myron Scholes, Robert Merton, and the last Fischer Black.

The details may be the devil's home stomping grounds, but in this work he has found his match—Danielle Lake of North Market Street Graphics, the project's copy editor. Thank you, Danielle.

For allowing me to plug in my laptop at *Once Upon A Tart*, thank you Jerome.

Contents

Volatility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$30.12
$27.25
Annualized Period
Expected CC Return 10.00 10.0000%
Median CC Return 5.500% 5.5000%
Standard Deviation 30.00 30.0000%
No Dividend 0.00 0.00
Current Asset Price 100.00
Investment Horizon 365
Draw Your Forecast
Simulate Price Change

Stock prices are volatile

1. To find the Clear button, mouse around at the bottom right of the screen. To zero out all the data fields, click on Clear three times.
2. To make anything you may have on screen go away, click two or three times on Hide.
3. Click on Simulate Price Change.

Let's say you're thinking about buying a stock.

Today it is trading at $100.00.

4. Tab to the Current Asset Price field. Enter 100.00. No dollar sign, please.

What's the price of this stock going to be one year from now?

We need a forecast.

Let's say that you believe in forecasts based on historical data.

You call up your broker and ask for the stock's historical performance. She says that, based on the past ten years, the stock has an expected return of 10% with a standard deviation of 30%. The standard deviation is a measure of the stock's volatility.

We'll talk a lot more about expected return and standard deviation later. For now, let's just enter the historical performance into the animation as a forecast.

5. Click in or tab to the Expected Continuously Compounded Return field and enter 10.00.
6. For Standard Deviation, enter 30.00.

Our investment horizon is one year or 365 days.

7. For Investment Horizon, enter 365.

Given this forecast, we can simulate a potential price path.

8. Click again on Simulate Price Change.

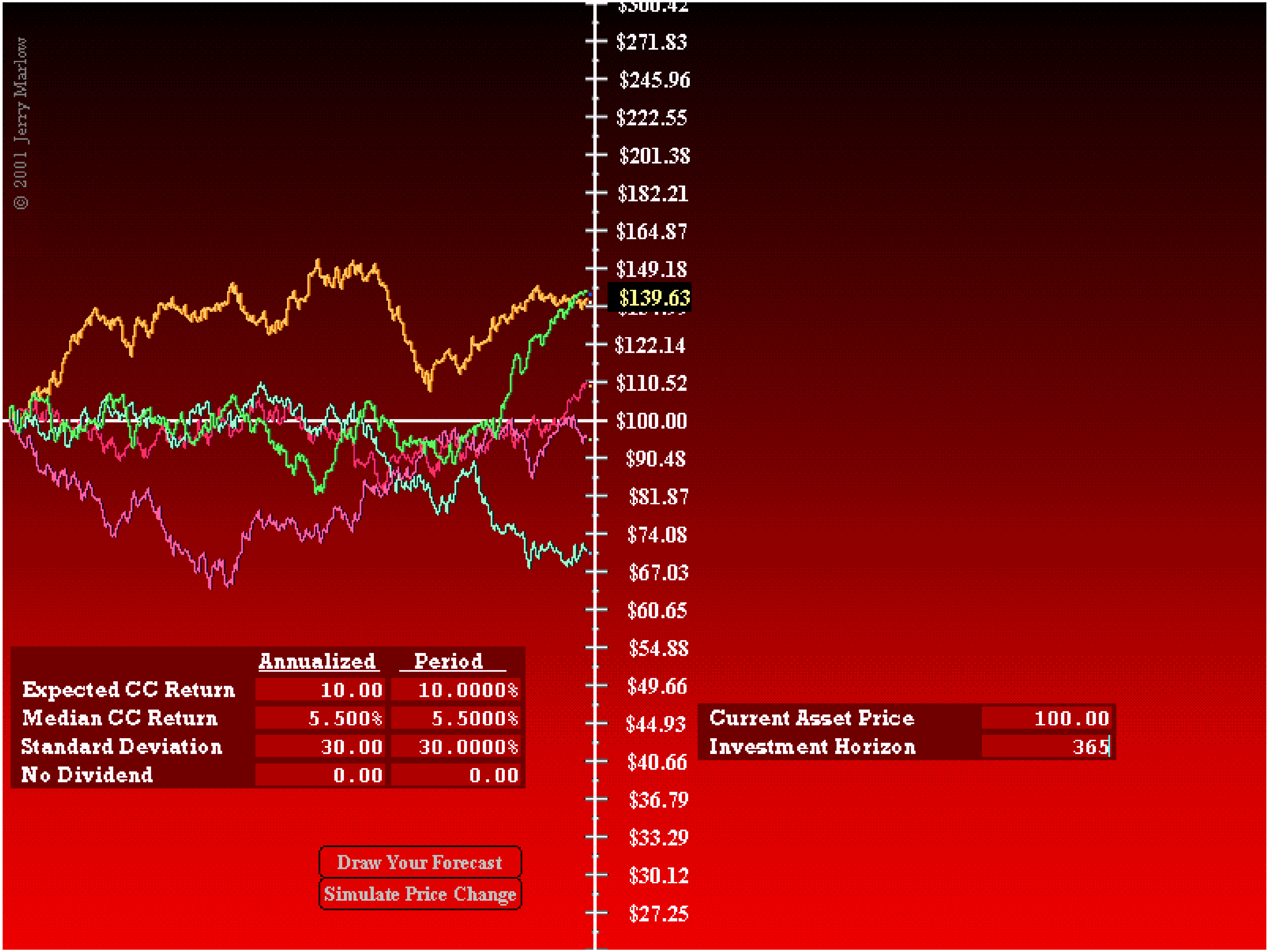
There. That looks pretty reasonable, doesn't it?

One year from today, the stock might be worth the value at the end of the price path.

We see that, over the course of the 365 days, the price keeps changing. What's more, the direction of price change keeps changing. That's volatility.

Is this the only potential price path we can get from this forecast?

No.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$36.79
$33.29
$30.12
$27.25
Annualized Period
Expected CC Return 10.00 10.0000%
Median CC Return 5.500% 5.5000%
Standard Deviation 30.00 30.0000%
No Dividend 0.00 0.00
Current Asset Price 100.00
Investment Horizon 365
Draw Your Forecast
Simulate Price Change

Volatility means that a stock's future price path is uncertain

9. Click again on Simulate Price Change.

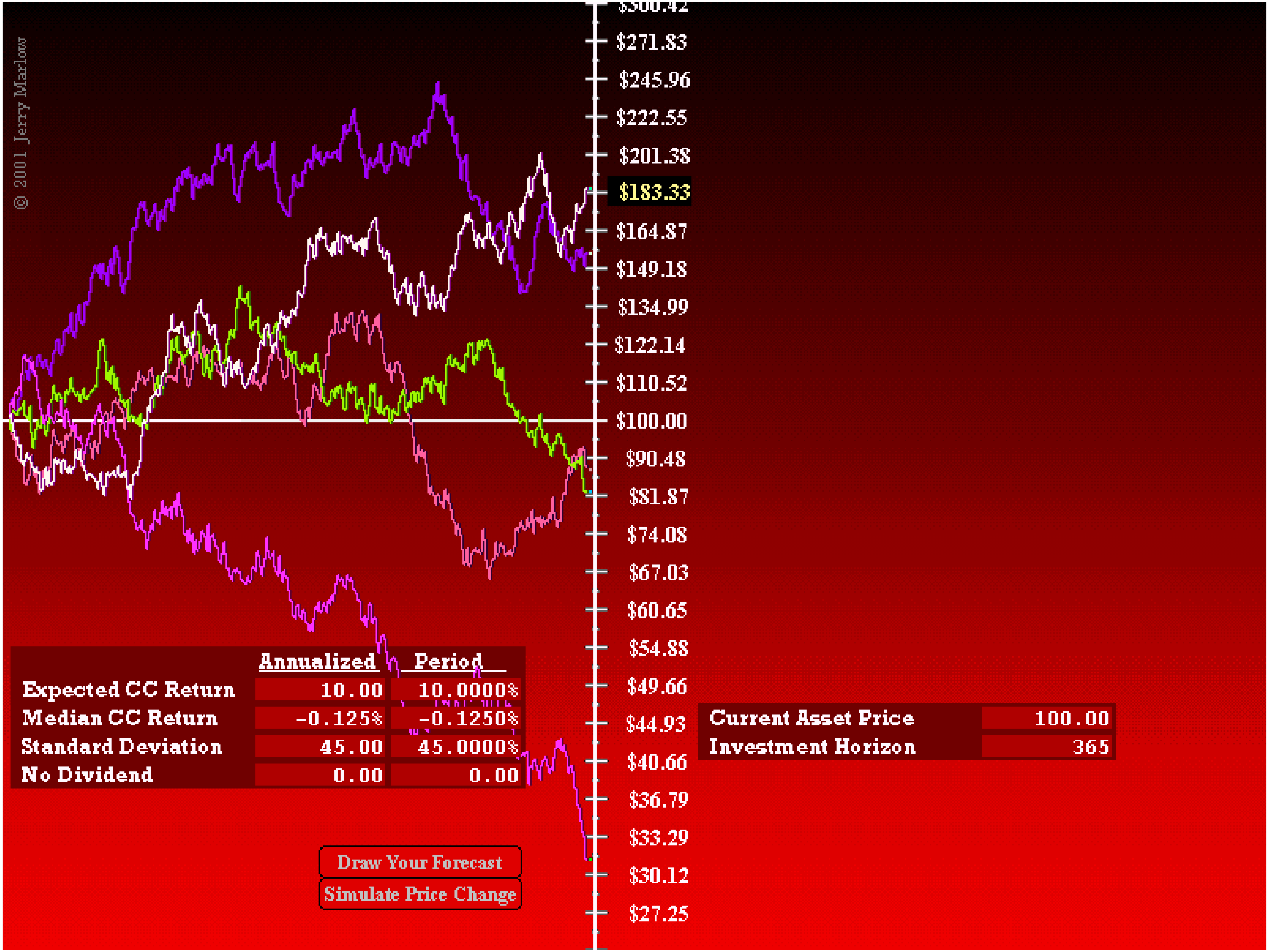
We get a different price path and a different outcome.

10. Click again on Simulate Price Change.

Same forecast. Different price path. Different outcome.

11. Click on Simulate Price Change a few more times. Each time, wait for the outcome.

All these price paths, all these outcomes, are allowed for in this one forecast. The volatility estimate allows for many different potential price paths.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$49.66
$44.93
$40.66
$36.79
$33.29
$30.12
$27.25
Annualized
Period
Expected CC Return 10.00 10.0000%
Median CC Return -0.125% -0.1250%
Standard Deviation 45.00 45.0000%
No Dividend 0.00 0.00
Current Asset Price 100.00
Investment Horizon 365
Draw Your Forecast
Simulate Price Change

The more volatile a stock, the more uncertain its future value

Let's see what happens to the potential price paths and outcomes if we change the volatility estimate.

If we put in a lower estimate of volatility, the animation draws a potential price path that has less volatility.

Let's see what a price path with a standard deviation of 20% looks like.

12. Click on Clear.
13. For Standard Deviation, enter 20.00.
14. Click on Simulate Price Change.

Not a straight line; but not a lot of volatility either.

15. Click on Simulate Price Change a few more times. Each time, wait for the outcome.

With a volatility estimate of 20%, the potential price paths and outcomes stay in a fairly narrow band.

If our forecast has a volatility of zero, what does that look like?

16. For Standard Deviation, enter 0.00.
17. Click on Simulate Price Change.

The price path is a straight line. No volatility at all.

18. Click again on Simulate Price Change.

With no volatility, we always get the same price path and the same outcome.

If the volatility estimate is significantly larger, say a standard deviation of 45%, what do the price paths look like?

19. For Standard Deviation, enter 45.0.
20. Click on Simulate Price Change five times. Each time, wait for the outcome.

We see that, the greater a stock's volatility, the more uncertain its future value.

End-of-period Asset Price	$144.40
- Strike Price	110.00
= Payoff	$34.40

Payoff	$34.40
- Cost of Option	5.50
= Profit	$28.90

Option Return	
Period CC Return	183.32%
Equivalent Simple Percent	525.38%
Annualized CC Return	743.46%
Equivalent Simple Percent	169258.92%

	Annualized	Period
Expected CC Return	10.00	2.4658%
Median CC Return	-0.125%	-0.0308%
Standard Deviation	45.00	22.3454%
No Dividend	0.00	0.00

Current Asset Price	100.00
Days to Expire (365 D/Y)	90
Call Strike Price	110.00
Call Option Price	5.50

Draw Your Forecast | Call

Simulate Price Change

If you buy an option—depending on the price path the stock takes— you can make a ton of money or you can lose it all

Right now we're working with a stock that has an expected return of 10% and a volatility estimate of 45%.

Let's look at what might happen if you bought a call option on this stock and held the option until expiration.

A call gives you the right to buy a stock at a pre-set price called the strike price.

Let's say you buy a call that expires in 90 days. It has a strike price of $110.00. The option sells for $5.50.

21. Click on Clear.
22. Click on Call.
23. For Days to Expire, enter 90.
24. For Call Strike Price, enter 110.00.
25. For Call Option Price, enter 5.50.

The yellow line represents the strike price at $110.00. For the call to be "in the money," the stock price has to be above the yellow line.

The distance from the yellow line to the green line represents the price of the option.

For you to make a profit from exercising the option, the stock price has to go above the green line.

Let's see what kind of results we might expect to get from buying this option and holding it until expiration.

26. Click on Simulate Price Change.

If the end-of-period price is above the strike price, the end-of-period price minus the strike price gives you your payoff.

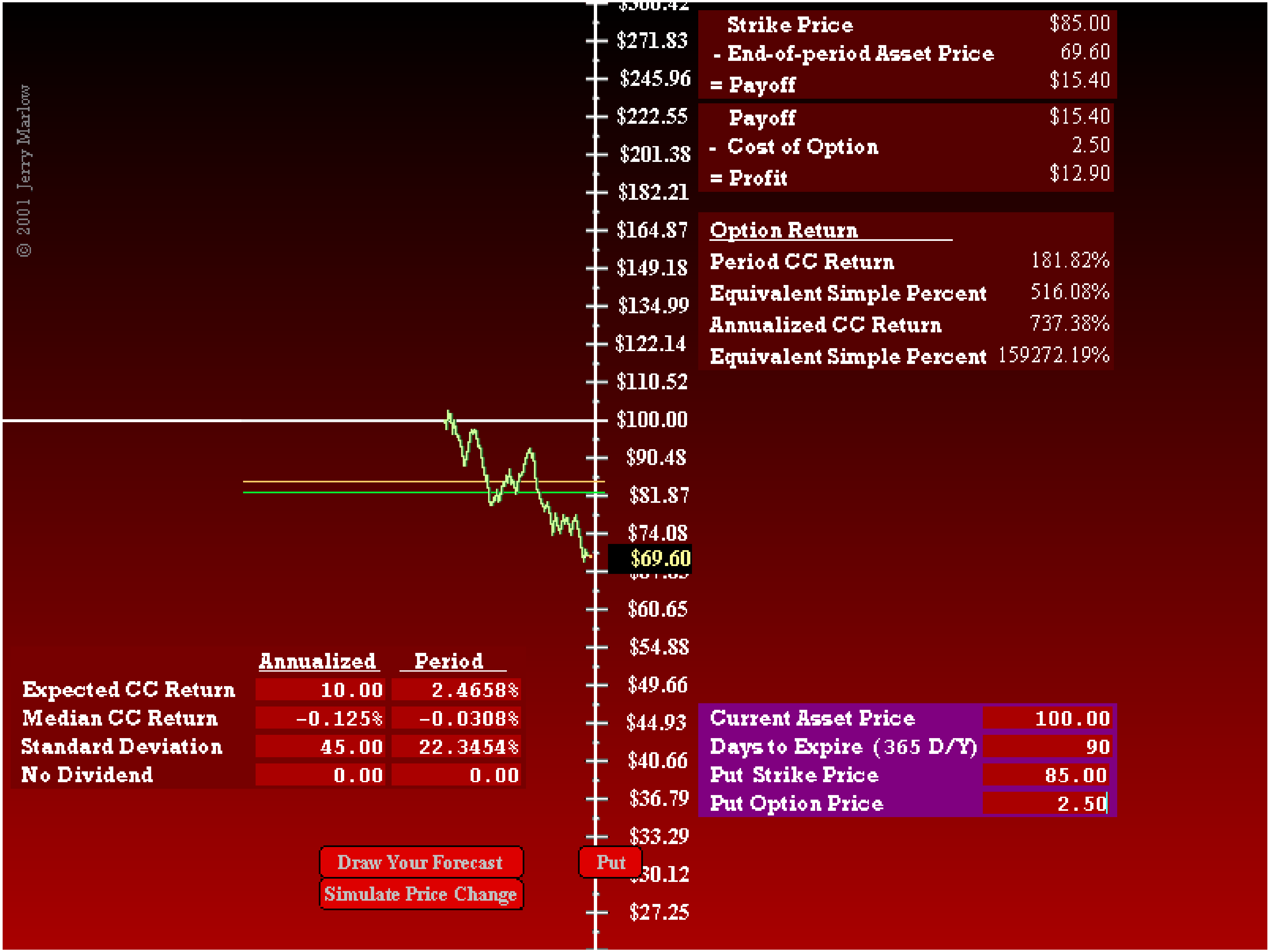
The payoff minus the cost of the option gives you your profit.

If the price path goes your way, by investing $5.50 in the option, you can earn a very high return.

If the end-of-period price is below the strike price, the payoff is zero. Your "profit" is negative, and you lose all the money you invested in the option.

27. Click on Simulate Price Change a few times. Each time, wait for the outcome. Keep clicking until you get a couple of winners.

You see that, depending on the price path the stock takes, you can earn wildly different rates of return.


© 2001 Jerry Marlow
Strike Price $85.00
- End-of-period Asset Price 69.60
= Payoff $15.40
Payoff $15.40
- Cost of Option 2.50
= Profit $12.90
Option Return
Period CC Return 181.82%
Equivalent Simple Percent 516.08%
Annualized CC Return 737.38%
Equivalent Simple Percent 159272.19%
$271.83
$245.96
$222.55
$201.38
$182.21
$164.87
$149.18
$134.99
$122.14
$110.52
$100.00
$90.48
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$74.08
$69.60
$60.65
$54.88
$49.66
$44.93
$40.66
$36.79
$33.29
$30.12
$27.25
Annualized Period
Expected CC Return 10.00 2.4658%
Median CC Return -0.125% -0.0308%
Standard Deviation 45.00 22.3454%
No Dividend 0.00 0.00
Current Asset Price 100.00
Days to Expire (365 D/Y) 90
Put Strike Price 85.00
Put Option Price 2.50
Draw Your Forecast
Simulate Price Change
Put

Instead of buying a call, you might buy a put. A put gives you the right to sell a stock at a pre-set price.

Let's say you buy a put on this same stock. The forecast remains the same.

The put expires in 90 days. It has a strike price of $85.00. It sells for $2.50.

28. Click on Clear.
29. Click on Put.
30. For Put Strike Price, enter 85.00.
31. For Put Option Price, enter 2.50.

To be in the money on a put, the stock price has to go *below* the yellow line. To earn a profit from exercising the option, the stock price has to go below the green line.

32. Click on Simulate Price Change a few times. Each time, wait for the outcome. Keep clicking until you get a couple of winners.

Chances are you'll see some breathtaking outcomes.

The option outcomes you've just simulated may suggest to you that options are nothing but risky gambles. They're not. Option prices and potential outcomes are completely rational. If they weren't, option sellers or buyers would have the opportunity to hedge their sales or purchases and earn riskless profits.

In *Black-Scholes Made Easy,* we go through a series of ideas and simulations that show the logic behind how options work. We keep the uncertainty of outcomes, but we show how it looks within a rational framework.

To understand European options, we use Black-Scholes. To understand American options, we add Black's approximation.

In the pages that follow, because it is easier to understand and to grasp intuitively, we begin by assuming that you, the investor, buy options and hold them until their maturity or expiration date. We look at potential outcomes of holding options until maturity. We look at their expected returns, probabilities of profit and probabilities of being in the money. We look at alternative ways of valuing or pricing options held to maturity.

After building our knowledge by looking at options held to maturity, we look at a more advanced topic: the circumstances in which it might prove advantageous to exercise an option prior to maturity. We discuss the different components of an option's value. We show how, under certain circumstances, exercising an option sooner may give a higher present value and expected return than exercising it later or at maturity.

Options sold on exchanges are of two basic types: European style and American style. European-style options can be exercised only at maturity. Most of the options on market indices are European style.

American-style options can be exercised at any time—up to and at maturity. Most stock options are American style.

The terms European and American have nothing to do with geographic location. They merely designate different types of option structures.

With an American-style option, you have all the rights you have with a European-style option plus the right to exercise it prior to maturity. Under certain circumstances, the right of early exercise has value. Hence, under those circumstances, an otherwise identical American option is more valuable than a European option.

The Black-Scholes model was designed and developed to value only European-style options—options held to maturity. It does not automatically look at the potential value of the right to early exercise.

To value American-style options, Fischer Black developed an approach now called Black's approximation. The approach includes a look at the potential value of early exercise.

To understand and value options held to maturity, we use the Black-Scholes model and the Black-Scholes assumptions. To understand and value American options, we add Black's approximation.

When we look at circumstances under which you might gain an advantage through early exercise, you will see, among other things, that never can you gain an advantage from exercising early a call option on an underlying that pays no dividends. Hence, on call options written on underlyings that pay no dividends, there is no difference in value between a European call and an American call. To value them, you can use Black-Scholes without Black's approximation.

The model most commonly used to value American-style options is the binomial pricing model. It automatically takes into account the possibility of gaining advantage through early exercise.

If you find the Black-Scholes animation of value and would like the author to develop an animated tutorial for the binomial pricing model, double-click on StayInTouch on the Black-Scholes Made Easy CD and send him an e-mail.

Every financial forecast is a probability distribution


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100%
90%
80%
70%
60%
50%
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$27.25
Annualized
Period
Expected CC Return 10.00 2.4658%
Median CC Return -0.125% -0.0308%
Standard Deviation 45.00 22.3454%
No Dividend 0.00 0.00
Current Asset Price 100.00
Investment Horizon 90
Draw Your Forecast
Simulate Price Change
Put
No Option

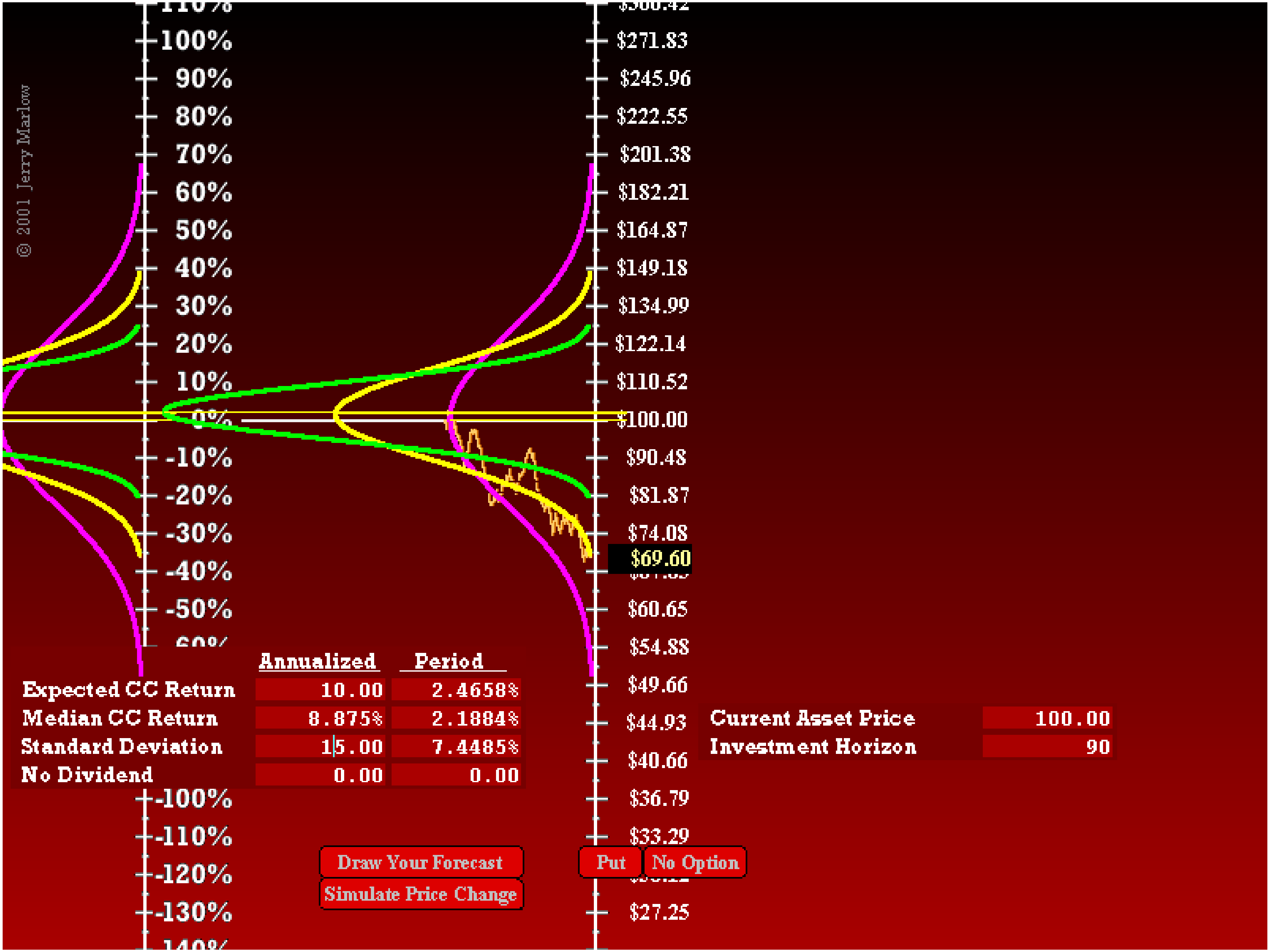
A forecast for a stock is a bell-shaped curve

Academics tell us, "Investing is about decision-making under conditions of uncertainty." This statement is true. But it is not terribly useful. A more useful construct is that every financial forecast is a probability distribution.

The forecast that you've been using to draw the price path you have on screen now is a probability distribution. You can draw it.

33. Click on No Option.
34. Click twice on Draw Your Forecast.

For the ninety-day investment horizon, the animation draws a forecast of return and a price forecast. Under the Black-Scholes assumptions, this is what the forecast for a stock looks like. It is the familiar bell-shaped curve.



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$40.66
$36.79
$33.29
$27.25
Annualized
Period
Expected CC Return 10.00 2.4658%
Median CC Return 8.875% 2.1884%
Standard Deviation 15.00 7.4485%
No Dividend 0.00 0.00
Current Asset Price 100.00
Investment Horizon 90
-100%
-110%
-120%
-130%
Draw Your Forecast
Simulate Price Change
Put
No Option

Different forecasts look different

Whenever you change a forecast's expected return or standard deviation, you get a different probability distribution.

35. For Standard Deviation, enter 10.00.
36. Click on Draw Your Forecast.

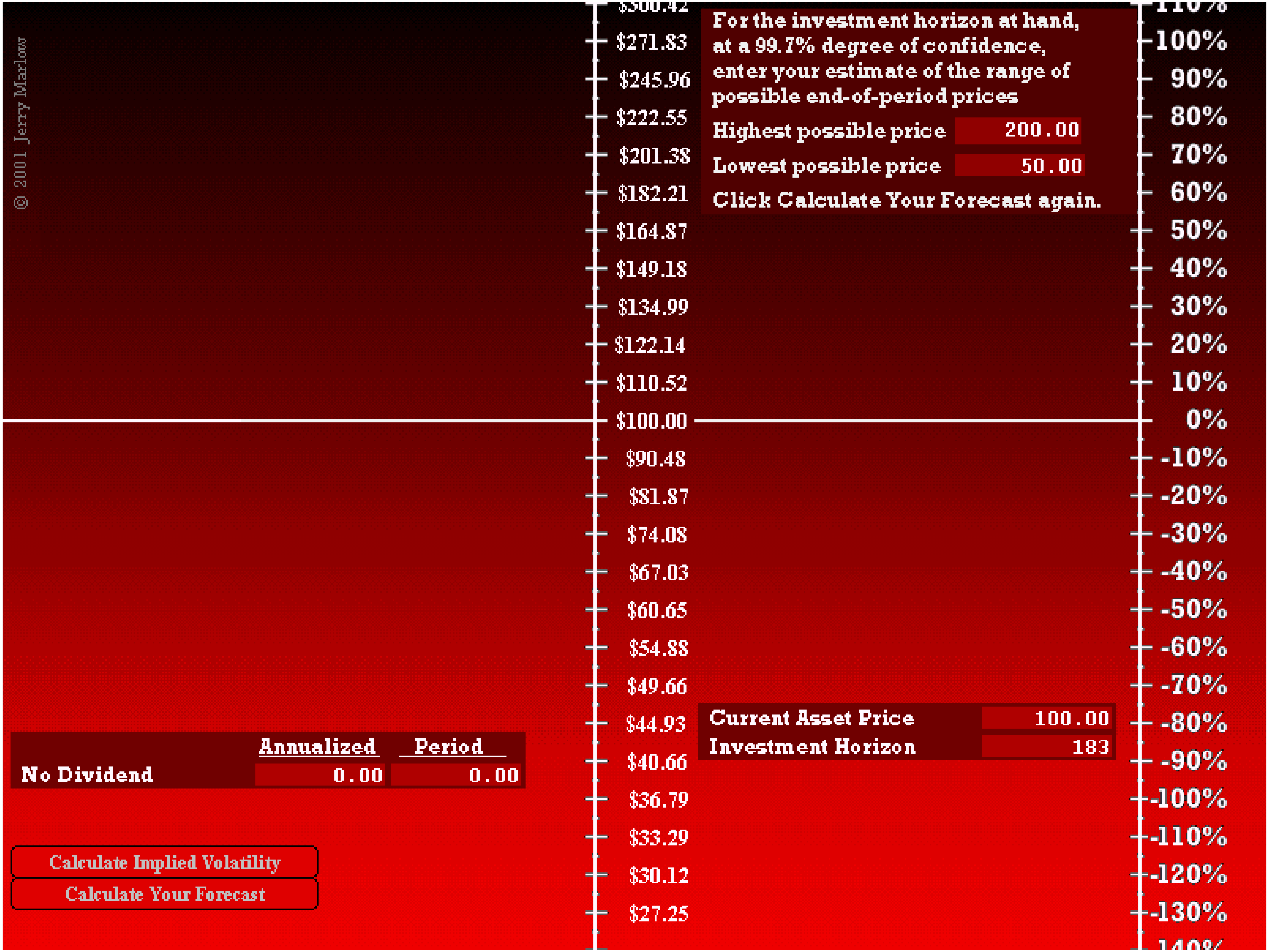
Notice the differences in placement and spread of the probability distributions.

37. For Standard Deviation, enter 30.00.
38. Click on Draw Your Forecast.

Most of what *Black-Scholes Made Easy* does is animate probability distributions. The animation combines the Black-Scholes assumptions, formulas, and ideas with what is known as structured Monte Carlo simulations.

Animations translate probability distributions into option prices and option prices into probability distributions. They translate probability distributions into potential price paths and price paths into illustrative payoffs.

The probability distributions we've been looking at are bell shaped. Later, we'll see that the probability distributions of options forecasts have a radically different shape.



© 2001 Jerry Marlow
For the investment horizon at hand, at a 99.7% degree of confidence, enter your estimate of the range of possible end-of-period prices
Highest possible price 200.00
Lowest possible price 50.00
Click Calculate Your Forecast again.
$271.83
$245.96
$222.55
$201.38
$182.21
$164.87
$149.18
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-50%
-60%
-70%
-80%
-90%
-100%
-110%
-120%
-130%
Current Asset Price 100.00
Investment Horizon 183
Annualized
Period
No Dividend 0.00 0.00
Calculate Implied Volatility
Calculate Your Forecast

You can translate your estimate of possible end-of-period prices into a forecast of expected return and uncertainty

Let's look at a crude way to translate your beliefs about a stock's future price performance into an elegant forecast of return and uncertainty.

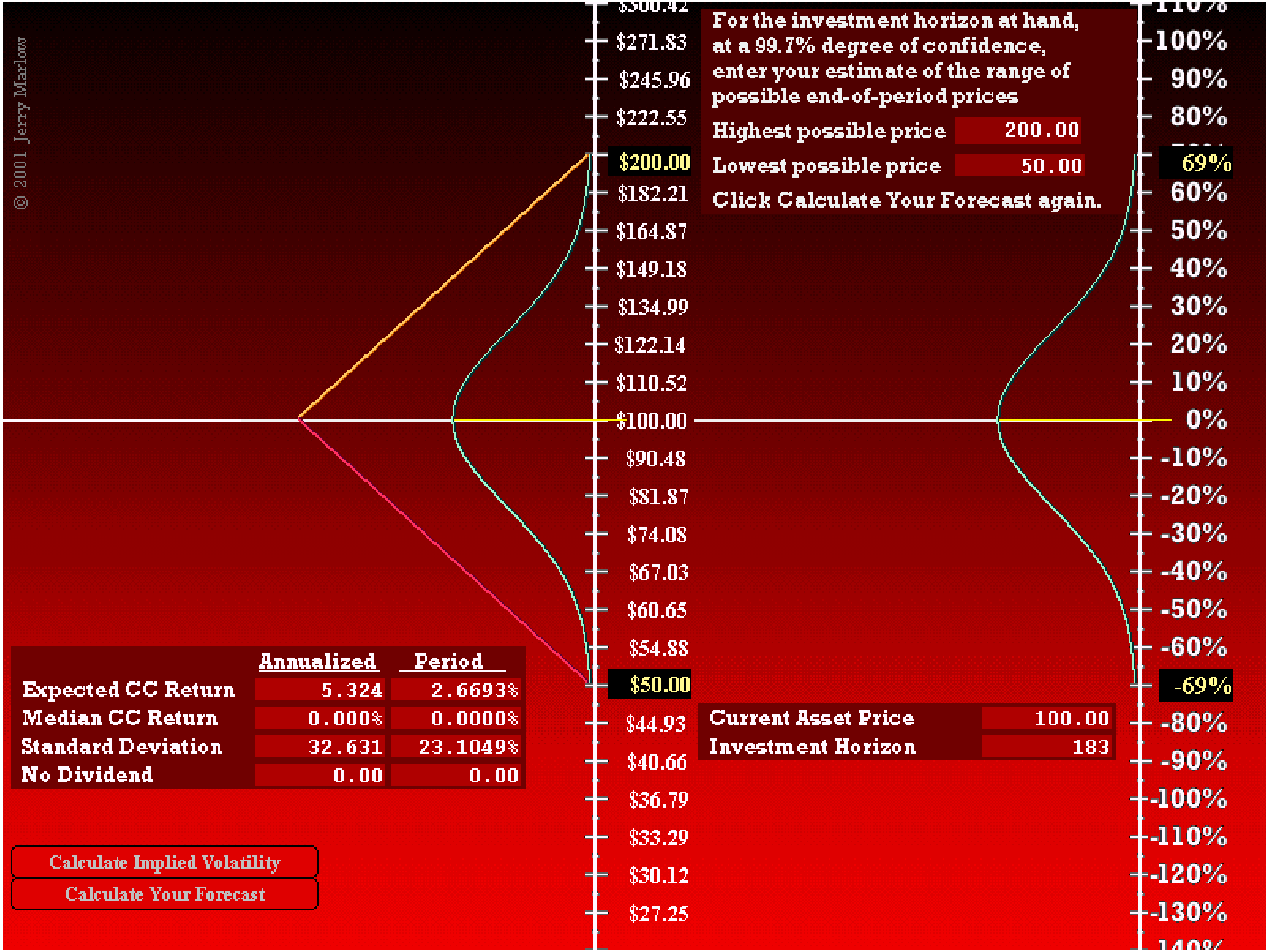
Let's say you're interested in a stock that today is trading at $100.00. The stock pays no dividends.

With a 99.7% degree of confidence, you believe that, over the next six months, the trading price of the stock may reach as high as $200.00 or it may fall as low as $50.00.

What is the forecast of return and uncertainty implicit in your beliefs about the price of this stock?

Let's see.

1. To zero out all the data fields, click three times on Clear.
2. To make everything on screen go away, click two or three times on Hide.
3. Click on Calculate Your Forecast.
4. Click in or tab to the field Current Asset Price.
5. Enter 100.00. (No $, please.)
6. For Investment Horizon, enter 183. That's 183 days for six months of a 365-day year.
7. For Highest possible price, enter 200.00. (No $, please.)
8. For Lowest possible price, enter 50.00.
9. Click Calculate Your Forecast.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$27.25
For the investment horizon at hand, at a 99.7% degree of confidence, enter your estimate of the range of possible end-of-period prices
Highest possible price 200.00
Lowest possible price 50.00
Click Calculate Your Forecast again.
100%
90%
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30%
20%
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0%
-10%
-20%
-30%
-40%
-50%
-60%
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-90%
-100%
-110%
-120%
-130%
Annualized Period
Expected CC Return 5.324 2.6693%
Median CC Return 0.000% 0.0000%
Standard Deviation 32.631 23.1049%
No Dividend 0.00 0.00
Current Asset Price 100.00
Investment Horizon 183
Calculate Implied Volatility
Calculate Your Forecast

The bell-shaped curve tells us that you are 99.7% certain that the investment outcome will be within the curve

The forecast the animation draws is a probability distribution—again the familiar bell-shaped curve.

To draw the forecast, the animation first calculates the continuously compounded rate of return (CCRR) that would be required for the current asset price of \$100.00 to reach \$200.00.

CCRR = ln(end-of-period price / starting price)
= ln(\$200.00 / \$100.00)
= 69.3%

The animation then calculates the continuously compounded rate of return that would be required for the current price of \$100.00 to fall to \$50.00.

CCRR = ln(end-of-period price / starting price)
= ln(\$50.00 / \$100.00)
= -69.3%

ln signifies taking the natural logarithm of a number. (In our discussion of geometric rates of return, we will review the meaning of the natural logarithm.)

The median or middle return of a normal, perfectly symmetrical probability distribution is equal to the average of the two extreme returns.

Median return = (69.3% + -69.3%)/2
= 0.0 / 2
= 0.0

Standard deviation is a measure of how much individual outcomes, on average, differ from a distribution's median outcome. At the 99.7% confidence level, a normal probability distribution spans six standard deviations. The range of this forecast for 183 days is from a return of 69.3% to a return of –69.3%. Therefore,

One standard deviation = (69.3% - -69.3%) / 6
= 138.6 / 6
= 23.1%

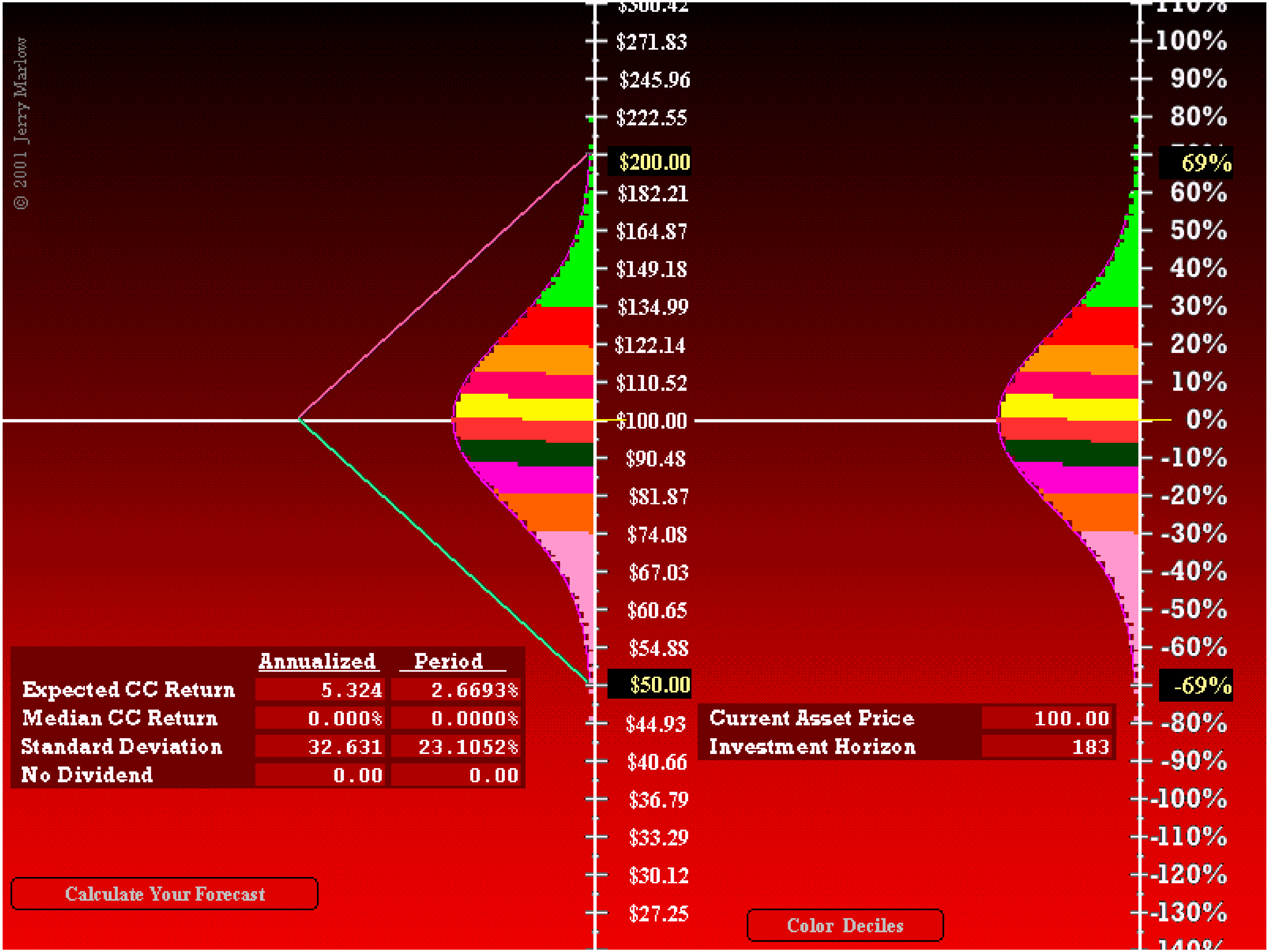
You see onscreen that the period standard deviation is 23.1049%.

These two numbers, the median return and the standard deviation, are all the information you need to draw a normal probability distribution.

What this forecast with a median of 0.0% and a standard deviation of 23.1% says is that, you are 99.7% confident that, 183 days from today, the price of the stock will be somewhere between \$50.00 and \$200.00. There is a 0.15% chance that the price will be greater than \$200.00 and a 0.15% chance that the price will be less than \$50.00.

You are 99.7% certain that the return will be somewhere between 69.3% and -69.3%. There is a 0.15% chance that the return will be greater than 69.3% and a 0.15% chance that the return will be less than –69.3%.

The bell shape of the probability distribution indicates that, at the end of the 183 days, prices are more likely to be near the middle of the distribution than they are to be at or near the extremes.



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-130%
Annualized Period
Expected CC Return 5.324 2.6693%
Median CC Return 0.000% 0.0000%
Standard Deviation 32.631 23.1052%
No Dividend 0.00 0.00
Current Asset Price 100.00
Investment Horizon 183
Calculate Your Forecast
Color Deciles

At the end of the investment period, there’s one chance in ten that the price will be in any given decile of the probability distribution

When you’re working with normal probability distributions, on average 68.3% of the outcomes fall within one standard deviation of the median. 95.4% of the outcomes fall within two standard deviations of the median. 99.7% of the outcomes fall within three standard deviations of the median.

(Later, we’ll look at how you can calculate a stock’s historical standard deviation from a history of its returns. We’ll also discuss the relationship among median return, standard deviation, and expected return.)

In case you’re not accustomed to working with standard deviations, the animation divides probability distributions into deciles.

10. Click on Color Deciles.

On average, 10% of the outcomes can be expected to fall within each decile or color band.

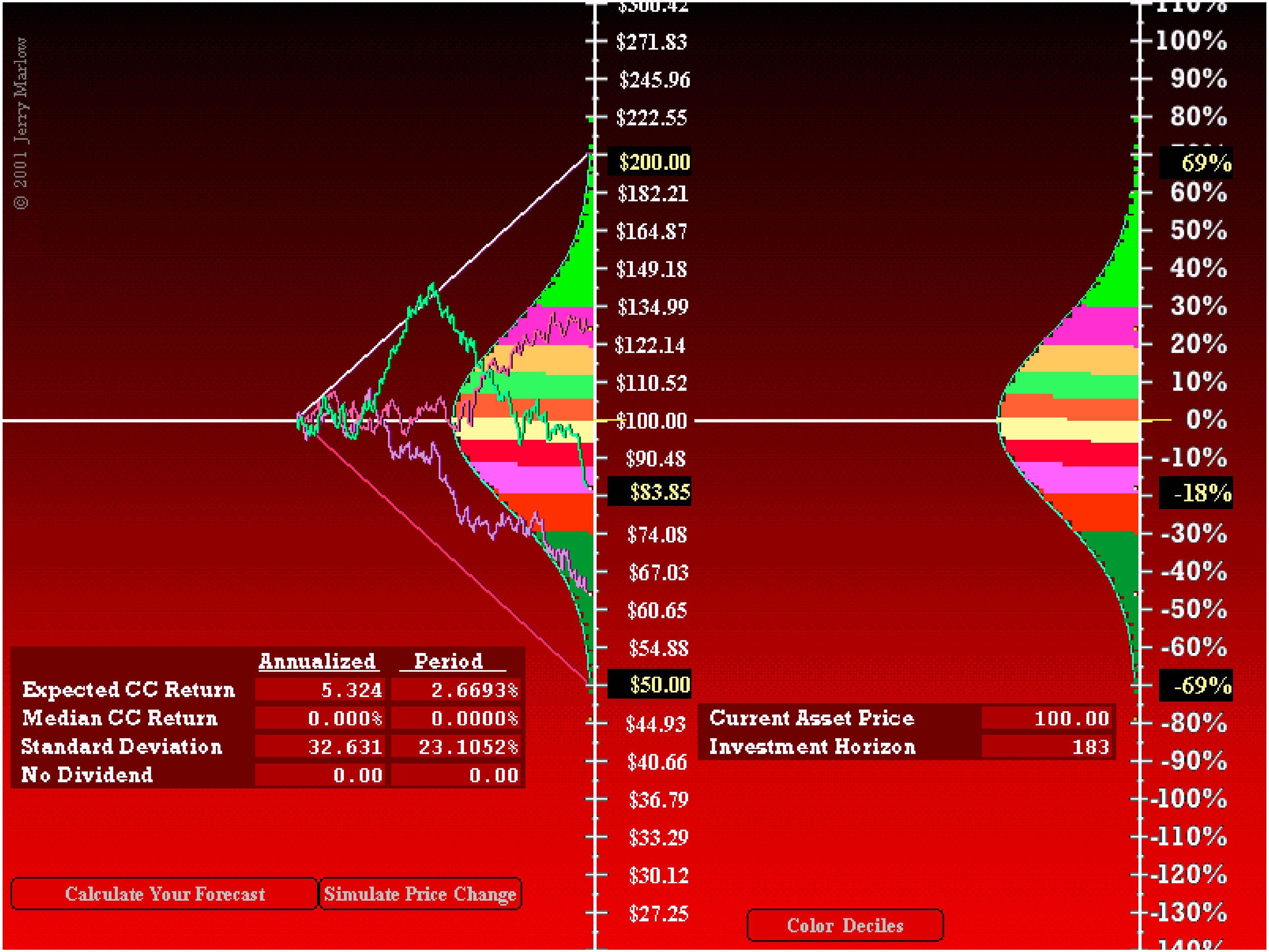
Or—to express the same idea a little bit differently—at the end of the investment period, there’s one chance in ten that the price will be in any given decile of the probability distribution.

Be kind to your brain

The animations make the topics we're exploring seem simple and easy. They're not. By the end of the animations—which you could go through in a few hours—you will have explored ideas, principles, and concepts that graduate business schools sometimes take months or even years to cover.

To become familiar with the animation routines, at the end of each section, go back through the steps in the section and play around with numbers of your own. Get a feel for how changing one variable changes the animation outcomes.

Before beginning a new section, check if the numbering of the animation steps continues from the previous section. If it does, the new section picks up where the previous one left off. If you've been playing around with your own numbers, before continuing, step through the previous section and enter the numbers from this book.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Annualized
Period
Expected CC Return 5.324 2.6693%
Median CC Return 0.000% 0.0000%
Standard Deviation 32.631 23.1052%
No Dividend 0.00 0.00
Current Asset Price 100.00
Investment Horizon 183
Calculate Your Forecast
Simulate Price Change
Color Deciles

You can translate a forecast into potential price paths

Using the median and standard deviation of a forecast, you can simulate potential price paths over the investment horizon.

Continuing with our example, we can simulate potential price paths over the investment horizon of 183 days.

11. Click on Simulate Price Change.

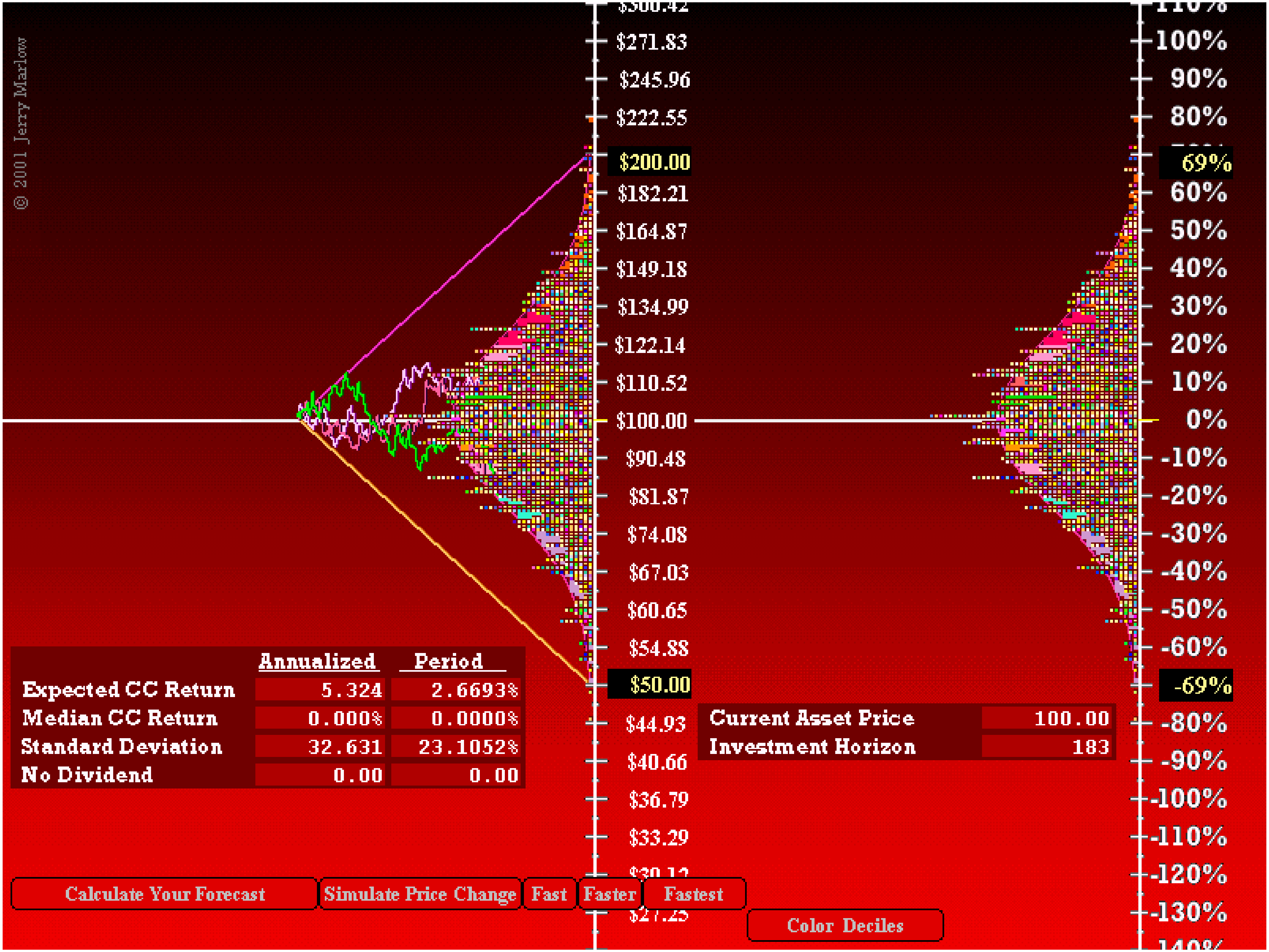
What you most likely see is a price path that moves around between the two extreme price paths and that ends up at a price somewhere in between $200.00 and $50.00.

12. Click on Simulate Price Change again.

Chances are you get a very different price path that ends up at a different end-of-period price.

13. Click on Simulate Price Change a few more times.

All these potential price paths are being generated from the same forecast. You see that having a forecast does not necessarily give you much information about what the end-of-period price or end-of-period return is going to be. As the academics say, "Investing is about decision-making under conditions of uncertainty."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Annualized Period
Expected CC Return 5.324 2.6693%
Median CC Return 0.000% 0.0000%
Standard Deviation 32.631 23.1052%
No Dividend 0.00 0.00
Current Asset Price 100.00
Investment Horizon 183
Calculate Your Forecast
Simulate Price Change
Fast
Faster
Fastest
Color Deciles

Structured Monte Carlo simulations show you the relationship between potential price paths and financial forecasts

Notice that each time you generate a price path, the animation tabulates each end-of-period price and each end-of-period return with a little square.

Instead of drawing the potential price path, you can just simulate the end-of-period price and end-of-period return.

14. Click on Fast just to the right of Simulate Price Change.

You see that the animation skips the potential price path and jumps to the end-of-period price and return.

15. Click on Faster to the right of Simulate Price Change.

The animation keeps simulating end-of-period prices and returns. To tabulate each, it adds new squares at the heights of the end-of-period price and return.

16. Click on Fastest.

The animation simulates enough end-of-period prices and end-of-period returns to fill in the areas of the probability distributions. Each pattern of squares creates a histogram that approximates the shape of the probability distribution or forecast.

If you've never run a structured Monte Carlo simulation before, congratulations. You just did. That's what generating, accumulating, and tabulating outcomes in this way is called.

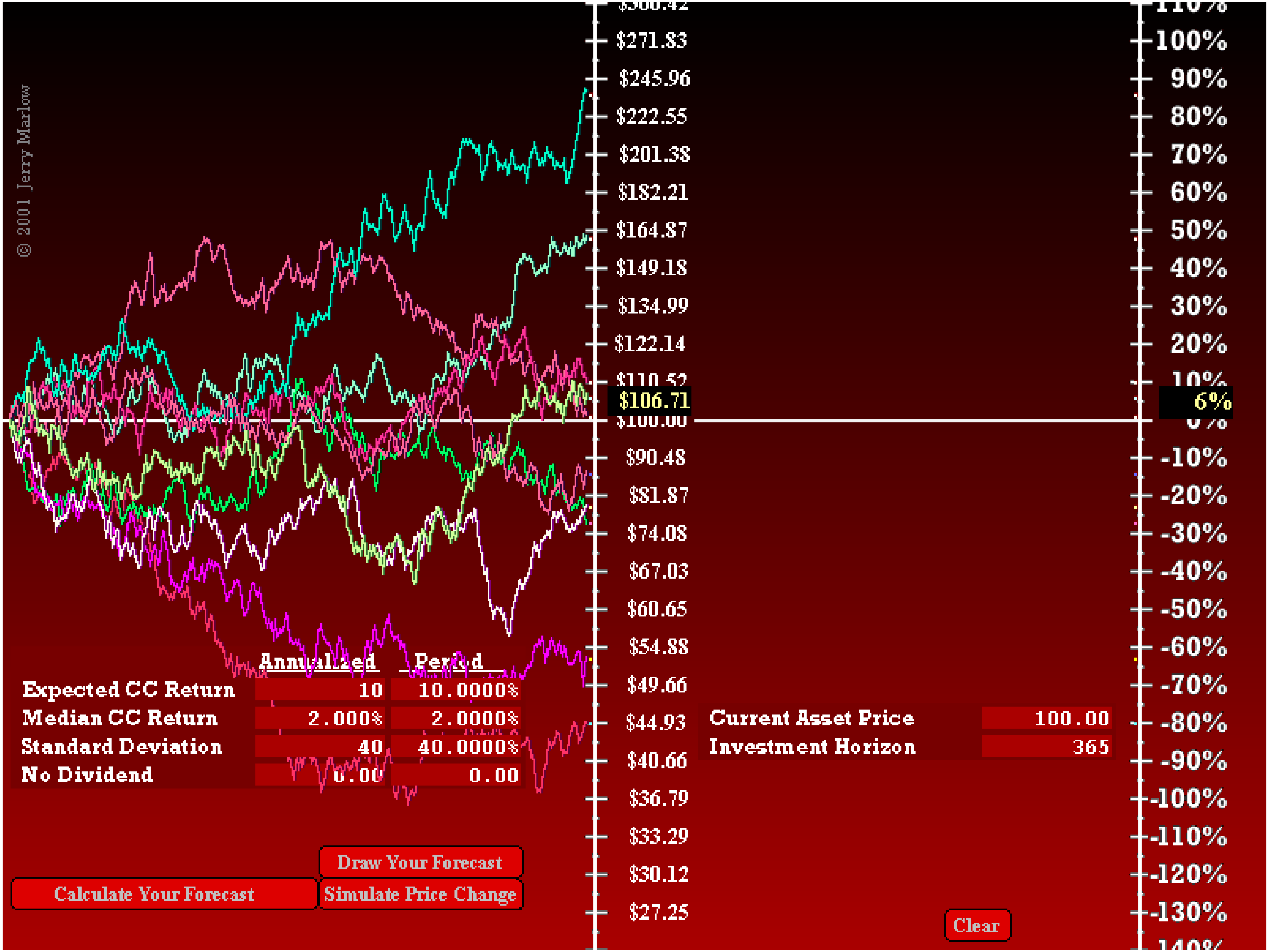
17. Click on Draw Your Forecast.

You see how closely the shape of the histogram matches that of your forecast.

Monte Carlo simulations
Here we're using a Monte Carlo simulation to help depict some of the relationships among a probability distribution, potential price paths, and a histogram. Investment firms often use structured Monte Carlo simulations to price derivatives whose payoffs are based on the interactions of many different financial instruments.

Using the probability distribution of each instrument and the correlations among them, the firms simulate potential outcomes for each instrument and tabulate the combined outcomes. Even though the outcome for each instrument is uncertain , a histogram of the combined outcomes shows the range and pattern of potential interactions.

Later, we'll use structured Monte Carlo simulations as one way to price options.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-90%
-100%
-110%
-120%
-130%
Annualized
Period
Expected CC Return
10
10.0000%
Median CC Return
2.000%
2.0000%
Standard Deviation
40
40.0000%
No Dividend
0.00
Current Asset Price
100.00
Investment Horizon
365
Draw Your Forecast
Calculate Your Forecast
Simulate Price Change
Clear

From its historical returns, you can calculate a stock's historical standard deviation

Standard deviation is a measure of how much individual returns, on average, differ from a normal distribution's mean or median return. (In a normal distribution, the mean and median are the same.)

We've been using median returns and standard deviations to simulate potential future returns of stocks. From historical returns, you can calculate a stock's historical mean return and standard deviation.

To demonstrate, first we generate a hypothetical sample of historical returns; then we find the sample's mean and standard deviation.

18. Click three times on Clear.
19. Tab to Expected CC Return. Enter 10.
20. For Standard Deviation, enter 40.
21. For Current Asset Price, enter 100.
22. For Investment Horizon, enter 365.
23. Click ten times on Simulate Price Change. Each time, record the return.

The calculations that follow use the returns from the screen capture above: -80%, 48%, 1%, -27%, -63%, -23%, 10%, 86%, -14% and 6%. (With small samples like this one, the sample mean and standard deviation are likely to differ noticeably from those used to generate the returns.)

1. Record returns as decimals and sum.

$$\begin{array}{r} -0.800 \\ 0.480 \\ 0.010 \\ -0.270 \\ -0.630 \\ -0.230 \\ 0.100 \\ 0.860 \\ -0.140 \\ \underline{0.060} \\ -0.560 \end{array}$$

2. Find the mean return.

$$\frac{-0.560}{10} = -0.056$$

3. Find the difference between each return and the mean.

$$\begin{array}{rl} -0.800 & -(-0.056) = -0.744 \\ 0.480 & -(-0.056) = \ \ 0.536 \\ 0.010 & -(-0.056) = \ \ 0.066 \\ -0.270 & -(-0.056) = -0.214 \\ -0.630 & -(-0.056) = -0.574 \\ -0.230 & -(-0.056) = -0.174 \\ 0.100 & -(-0.056) = \ \ 0.156 \\ 0.860 & -(-0.056) = \ \ 0.916 \\ -0.140 & -(-0.056) = -0.084 \\ 0.060 & -(-0.056) = \ \ 0.116 \end{array}$$

4. Square the differences and sum the squares. (Squaring gets rid of the minus signs.)

$$\begin{array}{rl} -0.744^2 & = 0.553536 \\ 0.536^2 & = 0.287296 \\ 0.066^2 & = 0.004356 \\ -0.214^2 & = 0.045796 \\ -0.574^2 & = 0.329476 \\ -0.174^2 & = 0.030276 \\ 0.156^2 & = 0.024336 \\ 0.916^2 & = 0.839056 \\ -0.084^2 & = 0.007056 \\ 0.116^2 & = \underline{0.013456} \\ & \ \ \ 2.134640 \end{array}$$

5. Divide the sum by the number of returns minus one. The result of this operation is the variance.

$$\frac{2.134640}{(10-1)} = 0.237182$$

6. To get the standard deviation, take the square root of the variance.

$$\sqrt{0.237182} = 0.487014$$

Standard deviation = 48.70%

(In practice, it's much easier to use the STDEV function in spreadsheet software such as Excel.)

Black-Scholes Assumptions
(Part I)

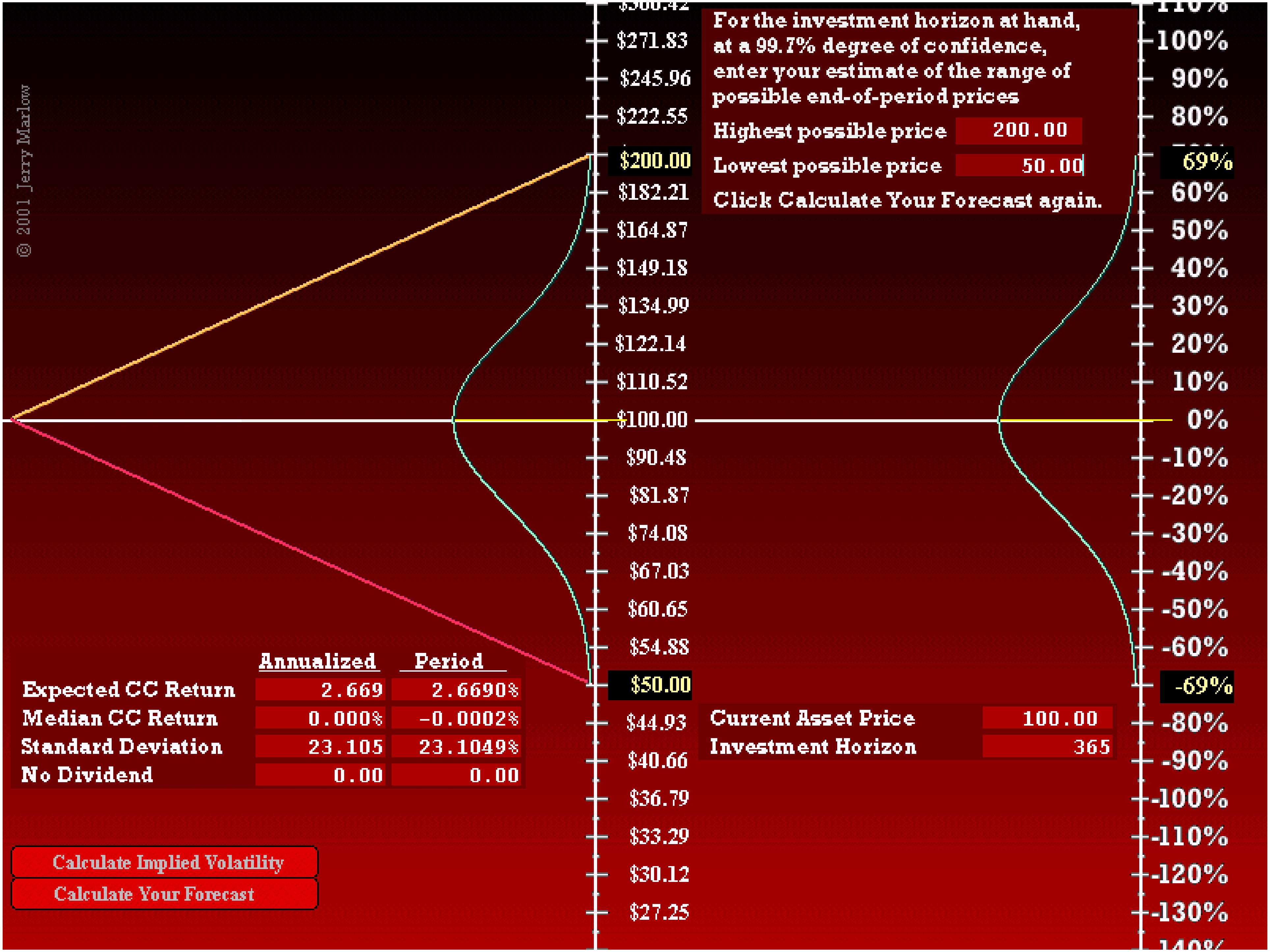
The animations you have drawn embody several of the Black-Scholes assumptions

Black-Scholes Options-Pricing Theory makes a number of assumptions about investors and the behavior of the financial markets. The animations that you thus far have drawn embody several of these assumptions:

- Stock returns expressed as geometric rates of return are normally distributed.
- Price changes are lognormally distributed.
- The potential price paths of a stock can be characterized by a geometric Brownian motion model.
- The volatility of a stock's price path is constant over the investment horizon.

The next few pages explain what these assumptions mean, how they are incorporated into the animations, and how they may differ from your customary way of thinking.

Black-Scholes makes several other assumptions about investors and the behavior of the financial markets. We will draw your attention to those as they come into play in the animations.



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-90%
-100%
-110%
-120%
-130%
Annualized
Period
Expected CC Return 2.669 2.6690%
Median CC Return 0.000% -0.0002%
Standard Deviation 23.105 23.1049%
No Dividend 0.00 0.00
Current Asset Price 100.00
Investment Horizon 365
Calculate Implied Volatility
Calculate Your Forecast

Assumption: Stock returns expressed as geometric or continuously compounded rates of return are normally distributed

Black-Scholes Options-Pricing Theory assumes that stock returns expressed as geometric rates of return are normally distributed. Accordingly, the animations and simulations draw return forecasts as normal distributions on axes labeled with geometric rates of return.

To see how geometric rates of return correspond to price changes, let's take a look at a $100 stock whose one-year forecast ranges from doubling in value to losing half its value.

1. To zero out all the data fields, Click three times on Clear.
2. To make everything on screen go away, click two or three times on Hide.
3. Click on Calculate Your Forecast.
4. Click in or tab to the field Current Asset Price.
5. Enter 100.00.
6. For Investment Horizon, enter 365.
7. For Highest possible price, enter 200.00.
8. For Lowest possible price, enter 50.00.
9. Click Calculate Your Forecast.

You see that a doubling of the stock price corresponds to a geometric rate of return of 69%. A halving of the price corresponds to a geometric rate of return of –69%.

Geometric rates of return are the same as continuously compounded rates of return. The idea is that, any percentage change in the stock price instantly creates a new base for future percentage changes. Consequently, when prices are going up, the base keeps getting bigger. When prices are going down, the base keeps getting smaller.

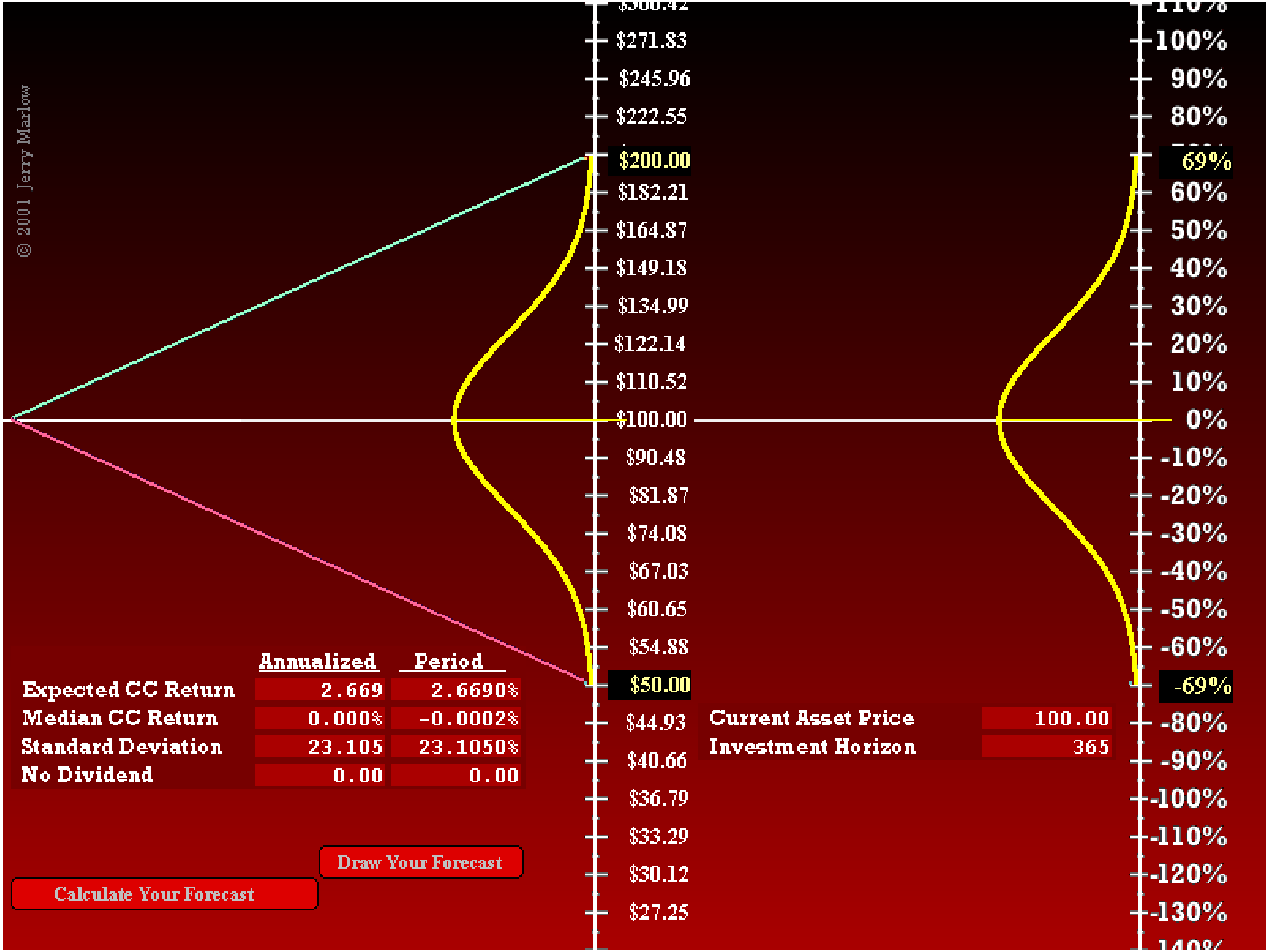
In general, the same principle is at work whenever you have compounding of gains or losses.

For example, a 10% gain compounded seven times produces almost double the initial value:

($100.00)(110%) = $110.00
($110.00)(110%) = $121.00
($121.00)(110%) = $133.10
($133.10)(110%) = $146.41
($146.41)(110%) = $161.05
($161.05)(110%) = $177.16
($177.16)(110%) = $194.87

A 10% loss compounded seven times reduces the initial value by slightly more than half:

($100.00)(90%) = $90.00
($90.00)(90%) = $81.00
($81.00)(90%) = $72.90
($72.90)(90%) = $65.61
($65.61)(90%) = $59.05
($59.05)(90%) = $53.14
($53.14)(90%) = $47.83



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Annualized
Period
Expected CC Return 2.669 2.6690%
Median CC Return 0.000% -0.0002%
Standard Deviation 23.105 23.1050%
No Dividend 0.00 0.00
Current Asset Price 100.00
Investment Horizon 365
Draw Your Forecast
Calculate Your Forecast

Assumption: Stock price changes are lognormally distributed

The exponential *e* serves as the base for natural logarithms and is used to compute continuously compounded or geometric rates of return.

A more formal look at continuous compounding shows how one arrives at the value of the exponential *e*.

In general, if we let *r* represent a period, simple rate of return and *n* represent the number of times the rate of return is compounded during the period, then the growth factor for the period is equal to

$(1+\frac{r}{n})^n$.

For example, for $100 with a period, simple rate of return of 70% compounded seven times,

$$(\$100)(1+\frac{r}{n})^n$$

gives:

$$(\$100)\,(1+\frac{.70}{7})^7$$
$$=(\$100)(1.10)^7$$
$$=(\$100)(1.9487)$$
$$=\$194.87$$

To take another example, for $100 with a period, simple rate of return of 100% compounded a million times,

$$(\$100)(1+\frac{r}{n})^n$$

gives:

$$(\$100)\,(1+\frac{1.00}{1{,}000{,}000})^{1{,}000{,}000}$$
$$=(\$100)(1.000001)^{1{,}000{,}000}$$
$$=(\$100)(2.718280)$$
$$=\$271.83$$

While a million is a large number of times to compound a rate of return, with continuous compounding, n becomes infinitely large.

The exponential *e* is defined as the value of $(1+\frac{r}{n})^n$ with *r* equal to 100% and *n* infinitely large. The value of $(1+\frac{1.00}{n})^n$ as *n* approaches infinity is approximately equal to 2.718281828459050. Hence, this number is the value of *e*, the base for natural logarithms.

Taking the natural log of an end-of-period price divided by the start-of-period price gives the continuously compounded rate of return:

ln ($200.00/$100.00) = 69.315%

ln ($271.83/$100.00) = 100%

ln ($50.00/$100.00) = -69.315%

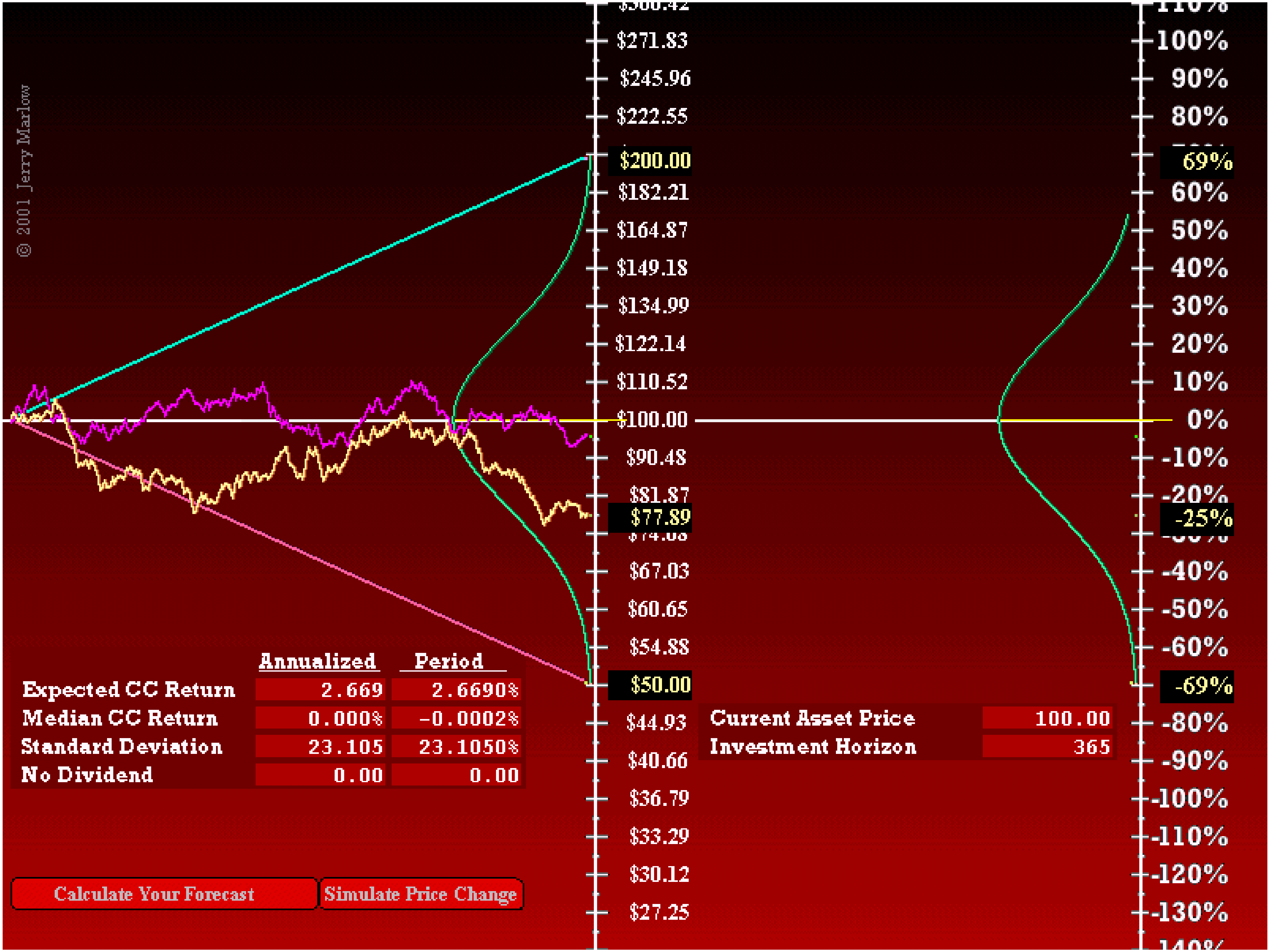
Multiplying a start-of-period price by the exponential raised to the power of the continuously compounded rate of return gives the end-of-period value:

($100.00)(exp(.69315)) = $200.00

($100.00)(exp(1.00)) = $271.83

($100.00)(exp(-.69315)) = $50.00

In the animations, the prices on the price axis make up a logarithmic or log scale. They correspond to the continuously compounded rates of return on the return axis. When we draw a normal distribution on the logarithmic price scale, we model price changes as being lognormally distributed.


© 2001 Jerry Marlow
$271.83
$245.96
$222.55
$200.00
$182.21
$164.87
$149.18
$134.99
$122.14
$110.52
$100.00
$90.48
$81.87
$77.89
$67.03
$60.65
$54.88
$50.00
$44.93
$40.66
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$33.29
$30.12
$27.25
100%
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60%
50%
40%
30%
20%
10%
0%
-10%
-20%
-25%
-40%
-50%
-60%
-69%
-80%
-90%
-100%
-110%
-120%
-130%
Annualized
Period
Expected CC Return 2.669 2.6690%
Median CC Return 0.000% -0.0002%
Standard Deviation 23.105 23.1050%
No Dividend 0.00 0.00
Current Asset Price 100.00
Investment Horizon 365
Calculate Your Forecast
Simulate Price Change

Assumption: The potential price paths of a stock can be characterized by a geometric Brownian motion model

Under the Black-Scholes assumptions, stock price paths follow a pattern described as geometric Brownian motion. Brownian motion is named for Robert Brown, a Scottish botanist.

Under his microscope, Brown, in 1827, noticed a "rapid oscillatory motion" of pollen grains suspended in water.

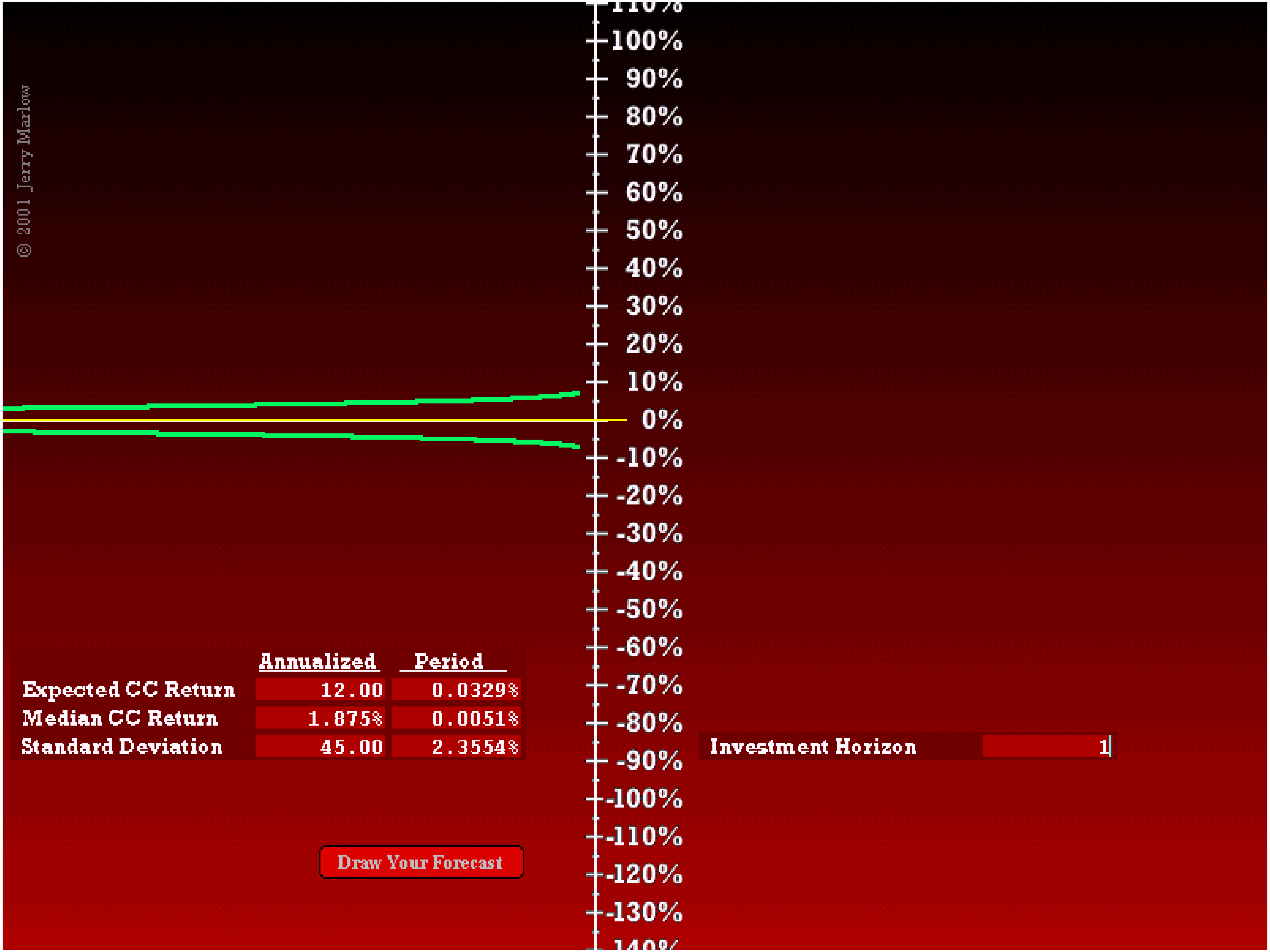
In 1905, Albert Einstein explained the motion as resulting from random differences between the pressures of molecules bombarding opposite sides of each pollen grain. The differences cause a grain constantly to wobble back and forth. Over a period of time, a grain tends to drift from its starting point. The probability of a grain moving a certain distance is characterized by a normal distribution.

Scientists and mathematicians have found Brownian motion to be a good model for many phenomena.

In Black-Scholes modeling, stock prices correspond to Brown's pollen grains. Bids and offers correspond to the molecules of water. Trades correspond to the collisions of the water molecules with pollen grains. Bids and offers, in a random way, knock the price of a stock up and down around the median return.

To take another look at potential price paths characterized by geometric Brownian motion:

10. Click a few times on Simulate Price Change.


© 2001 Jerry Marlow
100%
90%
80%
70%
60%
50%
40%
30%
20%
10%
0%
-10%
-20%
-30%
-40%
-50%
-60%
-70%
-80%
-90%
-100%
-110%
-120%
-130%
Annualized
Period
Expected CC Return
12.00
0.0329%
Median CC Return
1.875%
0.0051%
Standard Deviation
45.00
2.3554%
Investment Horizon
1
Draw Your Forecast

Assumption: The volatility of a financial asset's price path is constant over the life of the option

The characteristic volatilities of financial assets sometimes change. If the characteristic volatility changes during the life of an option, the value of the option changes.

The Black-Scholes model, however, does not allow for mid-life volatility changes. It assumes that the underlying's volatility will be constant over the life of the option.

To model the potential price paths of financial assets, the animation:

1. Calculates the one-day distribution of possible returns,
2. Takes a random sample of that distribution, and
3. Increases or decreases the start-of-day price by that amount.

The animation repeats this cycle of calculations for however many days are in the investment horizon.

To see what the one-day distribution of an asset's returns might look like:

1. Click three times on Clear.
2. Click two or three times on Hide.
3. Click on Draw Your Forecast.
4. For Investment Horizon, enter 1.
5. For Annualized Expected CC Return, enter 12.00.
6. For Standard Deviation, enter 45.00.
7. Click on Draw Your Forecast.

Working with Geometric or Continuously compounded Rates of Return


© 2001 Jerry Marlow
Portfolio Value 25.00
Cash Allocation 0.00
Risky Investment 25.00
Elapsed Time (Years)
Annualized CC Return
Equiv Simple Percent
$271.83
$245.96
$222.55
$201.38
$182.21
$164.87
$149.18
$134.99
$122.14
$110.52
$100.00
$90.48
$81.87
$74.08
$67.03
$60.65
$54.88
$49.66
$44.93
$40.66
$36.79
$33.29
$30.12
$27.25
500%
450%
400%
350%
300%
250%
200%
150%
100%
50%
0%
-50%
-100%
-150%
-200%
to
-600%
to
-Inf%
Annualized Period
Expected CC Return -20.00 -20.0000%
Median CC Return -20.000% -20.0000%
Standard Deviation 0.0 0.0000%
No Dividend 0.00 0.00
CC Risk-free Rate 0.0000%
Current Asset Price 100.00
Days to Expire (365 D/Y) 365
Call Strike Price 100.00
Call Option Price 25.00
Draw Your Forecast
Call
Simulate Investment
Calculate Black-Scholes Value
Draw Option Forecast
Analyze

Geometric or continuously compounded rates of return may not be your customary way of thinking

If you are accustomed to working with simple percentage, holding-period returns, you may find that continuously compounded rates of return give results at odds with your customary ways of thinking.

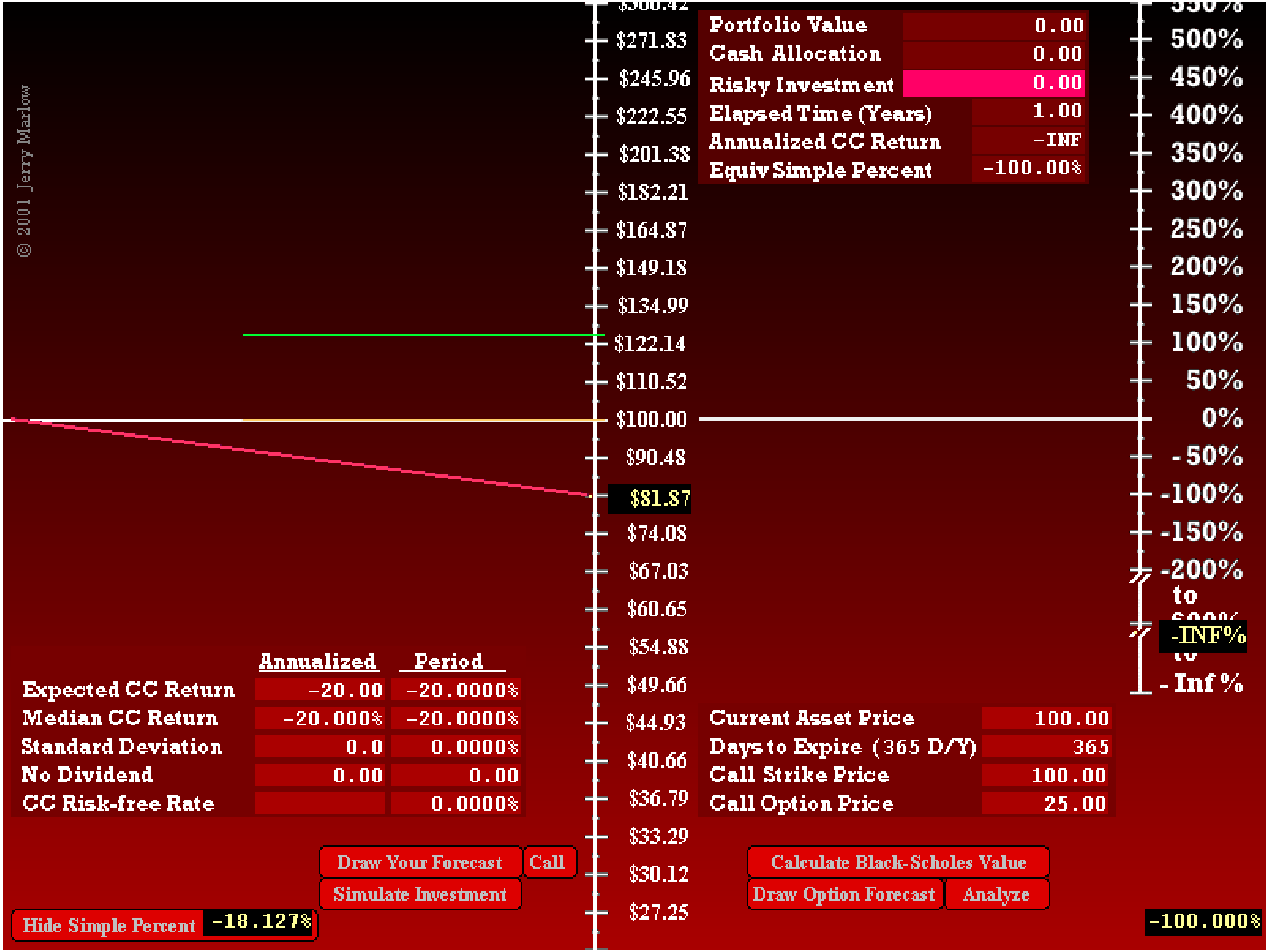
Let's look at a couple of examples of what some people find to be counter-intuitive aspects of working with geometric rates of return.

Let's pose the first example as a question: If you lose all your money, what is your continuously compounded rate of return?

Let's say you invest in an at-the-money call option on a stock. The option ends the period out of the money. What is the return on your investment?

Let's see.

1. To zero out all the data fields, Click three times on Clear.
2. To make everything on screen go away, click twice on Hide.
3. Click on Simulate Price Change.


© 2001 Jerry Marlow
Portfolio Value 0.00
Cash Allocation 0.00
Risky Investment 0.00
Elapsed Time (Years) 1.00
Annualized CC Return -INF
Equiv Simple Percent -100.00%
$271.83
$245.96
$222.55
$201.38
$182.21
$164.87
$149.18
$134.99
$122.14
$110.52
$100.00
$90.48
$81.87
$74.08
$67.03
$60.65
$54.88
$49.66
$44.93
$40.66
$36.79
$33.29
$30.12
$27.25
500%
450%
400%
350%
300%
250%
200%
150%
100%
50%
0%
-50%
-100%
-150%
-200%
to
-INF%
- Inf %
Annualized
Period
Expected CC Return -20.00 -20.0000%
Median CC Return -20.000% -20.0000%
Standard Deviation 0.0 0.0000%
No Dividend 0.00 0.00
CC Risk-free Rate 0.0000%
Current Asset Price 100.00
Days to Expire (365 D/Y) 365
Call Strike Price 100.00
Call Option Price 25.00
Draw Your Forecast
Call
Simulate Investment
Calculate Black-Scholes Value
Draw Option Forecast
Analyze
Hide Simple Percent -18.127%
-100.000%

When you lose all your money, you have a continuously compounded rate of return of negative infinity

4. For Current Asset Price, enter 100.00. (No $, please.)
5. Click on Draw Option Forecast.
6. Click on Call.
7. For Days to Expire, enter 365.
8. For Call Strike Price, enter 100.00.
9. For Call Option Price, enter 25.00.
10. Click on Invest.
11. For Portfolio Value, enter 25.00.
12. For Risky Investment, enter 25.00. When you have an option on screen, the risky allocation gets invested in the option.
13. Click on Simulate Investment.
14. For Expected CC Return, enter -20.00
15. For Standard Deviation, enter 0.0
16. Click on Simulate Investment.

The stock price goes to $81.87. The return on the stock is –20%.

The call option you bought finishes out of the money—meaning below the strike price. You lose all your money. The value of your portfolio goes to $0.00. What is your continuously compounded rate of return?

CCRR = ln(end-of-period value / starting value)
= ln($0.00 / $100.00)
= ln(0.00)
= -Infinity

When you lose all your money, you have a continuously compounded rate of return of negative infinity. As you will recall, when you're compounding negative returns, the base keeps getting smaller and smaller. With continuous compounding, to get to zero, the rate of return has to go to negative infinity.

To make it easier for you to keep straight the differences between holding period returns and continuously compounded or geometric rates of return, the animation almost always displays results in both forms.

17. Click on Show Simple Percent.

The animation does its calculations using geometric rates of returns; then it translates those results into equivalent simple percents.

The little box on the bottom left of your screen displays as an equivalent simple percent the return on the underlying. In this example, you see that, for the underlying's –20% return, the equivalent simple percent is –18.127%.

When you have an option on screen, the little box on the bottom right displays as an equivalent simple percent the return on the option.

For the option's return of negative infinity, the equivalent simple percent is –100.00%.

Whenever you enter a new value into the field labeled Expected CC Return, the box on the bottom left displays the equivalent simple percent of the period expected continuously compounded rate of return.


© 2001 Jerry Marlow
$271.83
$245.96
$222.55
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$182.21
$164.87
$149.18
$134.99
$122.14
$110.52
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$67.03
$60.65
$54.88
$49.66
$44.93
$40.66
$36.79
$33.29
$30.12
$27.25
Portfolio Value 200.00
Cash Allocation 100.00
Risky Investment 100.00
Elapsed Time (Years)
Annualized CC Return
Equiv Simple Percent
Annualized Period
Expected CC Return -20.00 -20.0000%
Median CC Return -20.000% -20.0000%
Standard Deviation 0.0 0.0000%
No Dividend 0.00 0.00
CC Risk-free Rate 20.00 20.0000%
Current Asset Price 100.00
Investment Horizon 365
Draw Your Forecast
Simulate Investment
Calculate Black-Scholes Value
Analyze

In your portfolio you have two $100 investments. One earns a continuously compounded return of 20%. The other earns a continuously compounded return of –20%. What is the continuously compounded return on your portfolio?

Here's another example of how working with geometric or continuously compounded rates of return can give counter-intuitive results.

Let's suppose that Jimmy Carter is president again. The continuously compounded risk-free rate of return is 20%. It's January 1. The value of your portfolio is $200.00. You're deciding how to allocate your portfolio between two investments: cash and a risky investment.

You allocate $100.00 to cash (meaning an instrument that will earn the risk-free rate of 20%).

You allocate $100.00 to a risky investment.

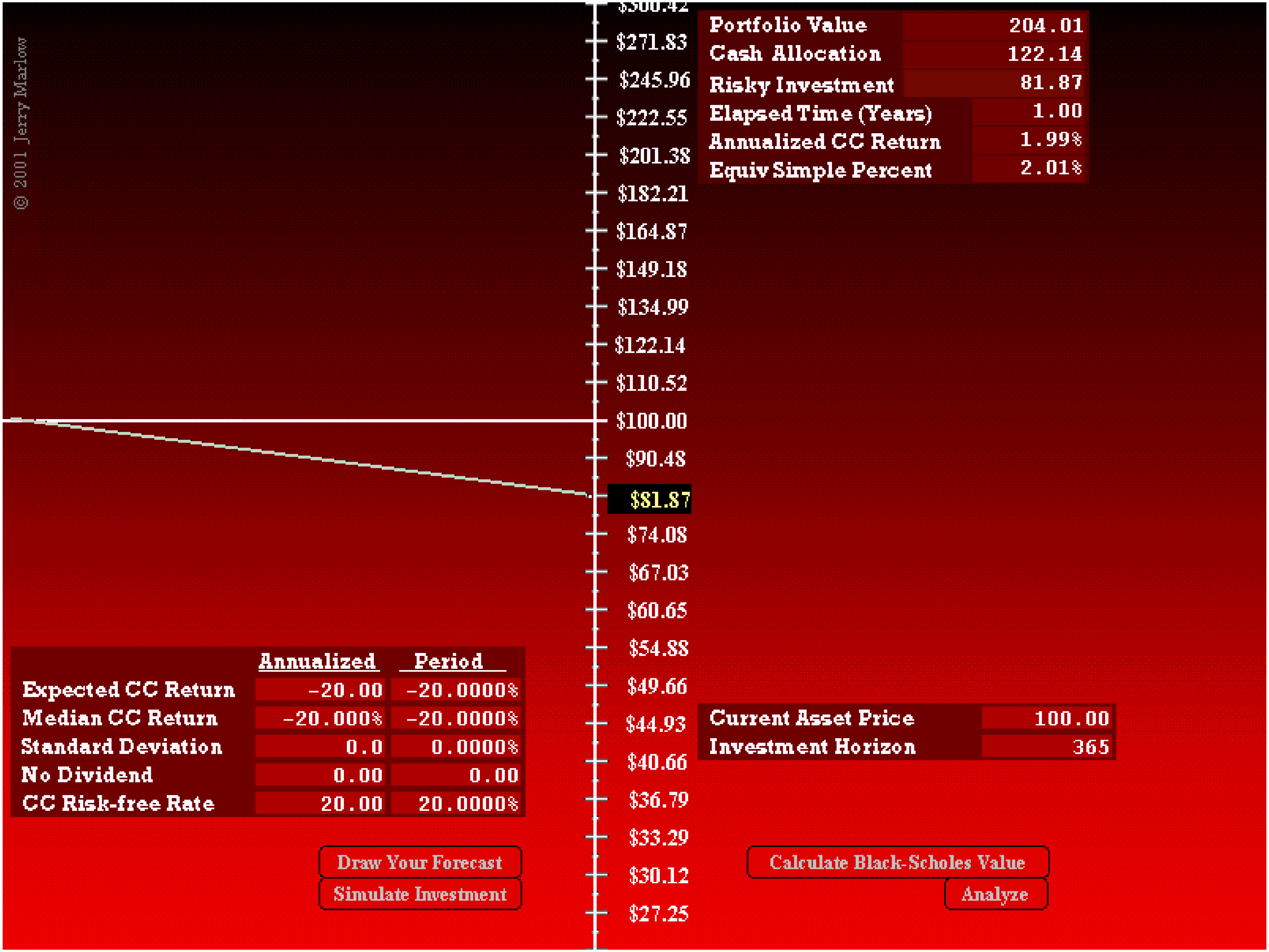
Over the course of the year, one investment earns a return of 20%. The other has a return of -20%.

At the end of the year, what is the return on your portfolio?

Jot down your answer here: ________

Let's see.

1. Click three times on Clear. (The third click clears the portfolio data fields.)
2. Click two or three times on Hide.
3. Click on Invest.
4. For Portfolio Value, enter 200.00. (No dollar sign, "$", please.)
5. For Risky Investment, enter 100.00.
6. For CC Risk-free Rate, enter 20.00
7. For Current Asset Price, enter 100.00.
8. For Investment Horizon, enter 365.
9. Click on Simulate Investment.
10. For Expected CC Return, enter -20.00.
11. For Standard Deviation, enter 0.00.
12. Click on Simulate Investment.


© 2001 Jerry Marlow
$300.42
$271.83
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$182.21
$164.87
$149.18
$134.99
$122.14
$110.52
$100.00
$90.48
$81.87
$74.08
$67.03
$60.65
$54.88
$49.66
$44.93
$40.66
$36.79
$33.29
$30.12
$27.25
Portfolio Value 204.01
Cash Allocation 122.14
Risky Investment 81.87
Elapsed Time (Years) 1.00
Annualized CC Return 1.99%
Equiv Simple Percent 2.01%
Annualized Period
Expected CC Return -20.00 -20.0000%
Median CC Return -20.000% -20.0000%
Standard Deviation 0.0 0.0000%
No Dividend 0.00 0.00
CC Risk-free Rate 20.00 20.0000%
Current Asset Price 100.00
Investment Horizon 365
Draw Your Forecast
Simulate Investment
Calculate Black-Scholes Value
Analyze

The portfolio return is not zero, but 1.99%

With a return of 20%, the cash allocation grows to \$122.14.

End-of-period value of the cash allocation = (\$100.00)(exp(.20))
= (\$100.00)(1.2214)
= \$122.14

With a return of –20%, the value of the risky investment falls to \$81.87.

End-of-period price of risky investment = (\$100.00)(exp(-0.20))
= (\$100.00)(0.8187)
= \$81.87

The value of the portfolio goes from \$200.00 to \$204.01 for a return of 1.99%, which is equivalent to a simple or holding-period return of 2.01%.

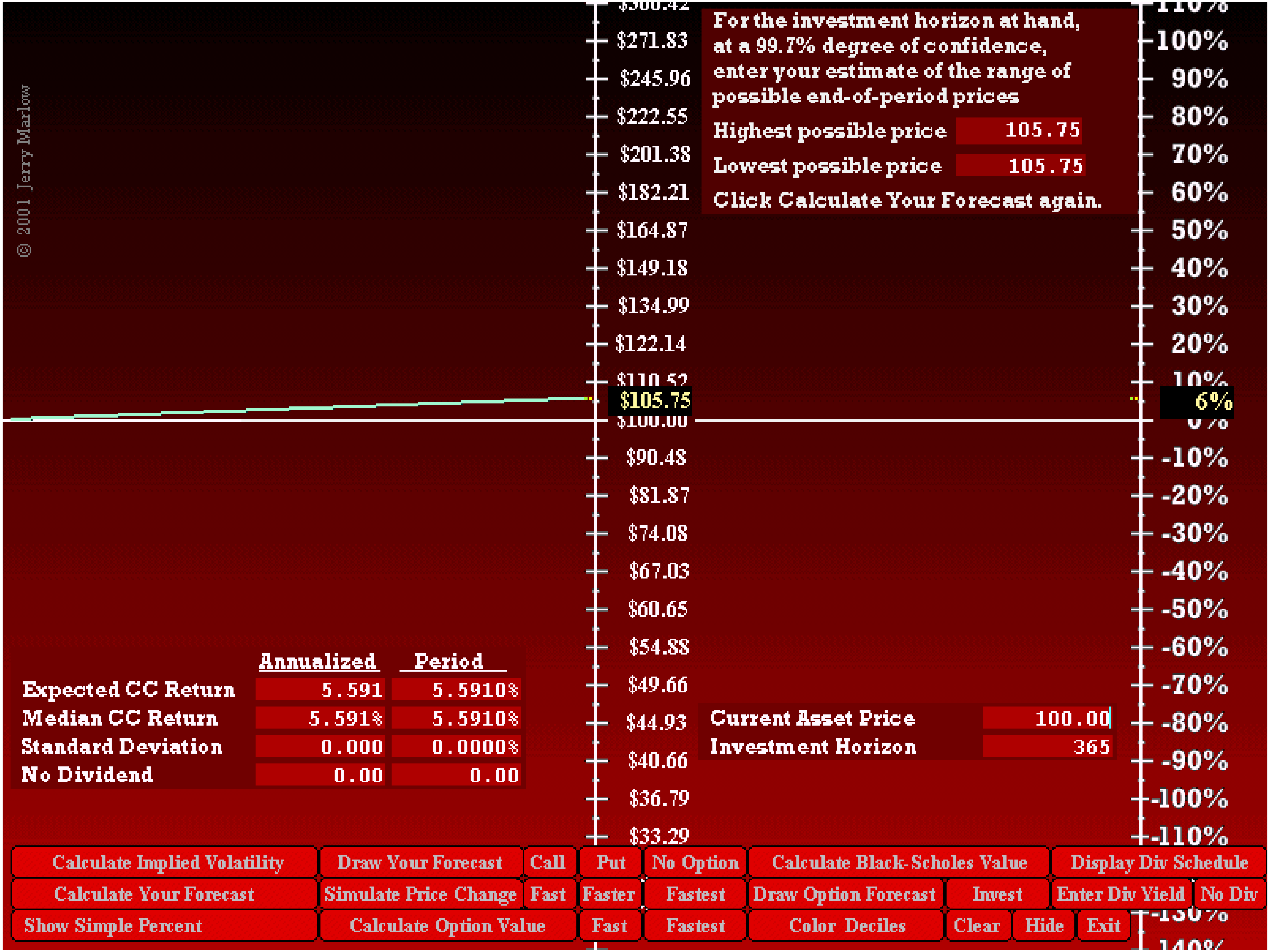
Portfolio CCRR = ln(End-of-period value / Starting value)
= ln((\$122.14 + \$81.87) / \$200)
= ln(\$204.01 / \$200.00)
= 1.99%

Equivalent simple percent = exp(CCRR) – 1.00
= exp(.0199) – 1.00
= 1.0201 – 1.00
= .0201
= 2.01%

When you have one investment that earns a return of 20% and another investment of equal initial value that has a return of –20%, the portfolio return is not zero, but 1.99%.

The average return is 1.99%.

When we compute an investment's expected return, you will see at work this same principle of the average return being greater than the middle return.


© 2001 Jerry Marlow
For the investment horizon at hand, at a 99.7% degree of confidence, enter your estimate of the range of possible end-of-period prices
Highest possible price 105.75
Lowest possible price 105.75
Click Calculate Your Forecast again.
$271.83
$245.96
$222.55
$201.38
$182.21
$164.87
$149.18
$134.99
$122.14
$105.75
$90.48
$81.87
$74.08
$67.03
$60.65
$54.88
$49.66
$44.93
$40.66
$36.79
$33.29
100%
90%
80%
70%
60%
50%
40%
30%
20%
6%
-10%
-20%
-30%
-40%
-50%
-60%
-70%
-80%
-90%
-100%
-110%
Annualized
Period
Expected CC Return 5.591 5.5910%
Median CC Return 5.591% 5.5910%
Standard Deviation 0.000 0.0000%
No Dividend 0.00 0.00
Current Asset Price 100.00
Investment Horizon 365
Calculate Implied Volatility
Draw Your Forecast
Call
Put
No Option
Calculate Black-Scholes Value
Display Div Schedule
Calculate Your Forecast
Simulate Price Change
Fast
Faster
Fastest
Draw Option Forecast
Invest
Enter Div Yield
No Div
Show Simple Percent
Calculate Option Value
Fast
Fastest
Color Deciles
Clear
Hide
Exit

To convert simple interest rates to continuously compounded rates of return, use Calculate Your Forecast

As a practical matter, you may need to convert simple annual interest rates to continuously compounded rates of return. You can do so easily with Calculate Your Forecast.

Let's say that the annual risk –free interest rate is 5.75%. You want to know what that is as a continuously compounded rate of return.

1. Click three times on Clear.
2. Click twice on Hide.
3. Click on Calculate Your Forecast.
4. For Current Asset Price, enter 100.00.
5. For Investment Horizon, enter 365.
6. For Highest possible price, enter the value of $100.00 a year from today. In our example, that's 105.75.
7. For Lowest possible price, also enter the value of $100.00 a year from today. In our example, 105.75.
8. Click on Calculate Your Forecast.

The continuously compounded rate of return appears in the Expected CC Return box. In our example, it's 5.591%.


© 2001 Jerry Marlow
$271.83
$245.96
$222.55
$201.38
$182.21
$164.87
$149.18
$134.99
$122.14
$110.52
$100.00
$90.48
$81.87
$74.08
$67.03
$60.65
$54.88
$49.66
$44.93
$40.66
$36.79
$33.29
Portfolio Value 98630.86
Cash Allocation 98630.86
Risky Investment 0.00
Elapsed Time (Years) 0.25
Annualized CC Return -5.59%
Equiv Simple Percent -5.44%
Annualized Period
Expected CC Return 0.0000%
Median CC Return 0.000% 0.0000%
Standard Deviation 0.0000%
No Dividend 0.00 0.00
CC Risk-free Rate -5.591 -1.3786%
Current Asset Price 100.00
Investment Horizon 90
Calculate Implied Volatility
Draw Your Forecast
Call
Put
No Option
Calculate Black-Scholes Value
Display Div Schedule
Calculate Your Forecast
Simulate Investment
Fast
Faster
Fastest
Draw Option Forecast
Analyze
Enter Div Yield
No Div
Show Simple Percent
Calculate Option Value
Fast
Fastest
Color Deciles
Clear
Hide
Exit

To find the present value of a future dollar amount, use "Invest" and "Simulate Investment"

When you are figuring out your option strategies, you may want to know what amount of money you need to invest today at the risk-free rate of return to reach a target value on some future date. In other words, you may want to know the present value of a future value discounted at the risk-free rate.

The concepts of present value and future value occur in common, everyday financial transactions. You can multiply a present value by 1 plus the simple interest rate to get a future value.

For example, if you buy a certificate of deposit (CD) today for $100 and it pays 10% simple interest, its future value one year from today is $110.

($100)(1.10) = $110.00.

Or you can go the other way. To get a present value, you can divide the future value by 1 + the simple interest rate.

If a CD will be worth $110 a year from today and it pays 10% simple interest, its present value is $100.

$110.00/1.10 = $100.00.

Financial professionals would describe this operation as "discounting the future value at the simple interest rate."

In the Black-Scholes world, you are more likely to want to discount a future value at the continuously compounded risk-free rate. Using the animation, the calculation is easy to make.

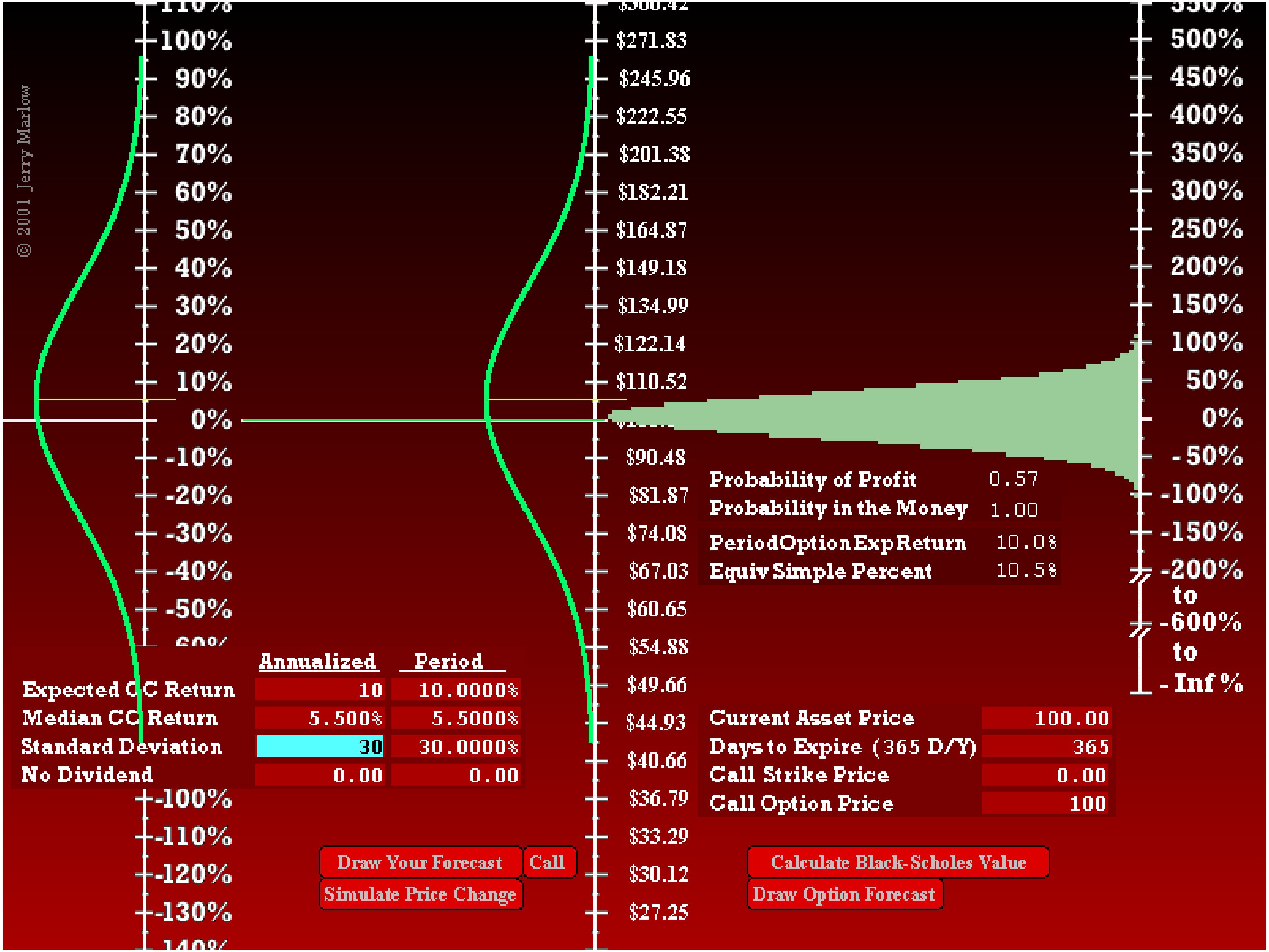
1. Click three times on Clear.
2. Click twice on Hide.
3. Click on Invest.
4. Click on Simulate Investment.
5. For Current Asset Price, enter 100.00.
6. For Investment Horizon, enter the number of days until the target date, for example, 90.
7. For CC Risk-free Rate, enter the continuously compounded risk-free rate of return with a minus (-) sign. For example, if the relevant rate is 5.591%, enter -5.591.
8. For Portfolio Value, enter the amount you wish to have on your target date in the future. As an example, 100000. (No commas, please.)
9. Click on Simulate Investment.

In 90 days, $100,000 at a continuously compounded rate of return of -5.591% would decrease to $98,630.86. That means that, at a rate of 5.591%, in 90 days, $98,630.86 would grow to $100,000.

To verify,

10. In the CC Risk-free Rate field, delete the minus sign in front of -5.591.
11. Click on Simulate Investment.

Expected Return



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100%
90%
80%
70%
60%
50%
40%
30%
20%
10%
0%
-10%
-20%
-30%
-40%
-50%
$271.83
$245.96
$222.55
$201.38
$182.21
$164.87
$149.18
$134.99
$122.14
$110.52
$90.48
$81.87
$74.08
$67.03
$60.65
$54.88
$49.66
$44.93
$40.66
$36.79
$33.29
$30.12
$27.25
500%
450%
400%
350%
300%
250%
200%
150%
100%
50%
0%
-50%
-100%
-150%
-200%
to
-600%
to
-Inf %
Probability of Profit 0.57
Probability in the Money 1.00
PeriodOptionExpReturn 10.0%
Equiv Simple Percent 10.5%
Annualized
Period
Expected CC Return 10 10.0000%
Median CC Return 5.500% 5.5000%
Standard Deviation 30 30.0000%
No Dividend 0.00 0.00
-100%
-110%
-120%
-130%
Current Asset Price 100.00
Days to Expire (365 D/Y) 365
Call Strike Price 0.00
Call Option Price 100
Draw Your Forecast
Call
Simulate Price Change
Calculate Black-Scholes Value
Draw Option Forecast

An investment's expected return is the average of all the returns in its probability distribution

Many people casually assume that, if you make an investment that has a certain expected return, you can expect to earn that return. This is not the case. Expected return is a statistical concept that has a precise mathematical definition.

An investment's expected return is the average of all the returns in its probability distribution. What's more, when we assume that an investment's geometric rate of return is normally distributed, the average of the returns is not the same as the middle or median return.

In the simple portfolio example we looked at above, we saw that the average of returns of 20% and -20% is not 0, but 1.99%. Similarly, the average of all the returns in a probability distribution is not the median. It's greater than the median.

The animation is designed to show how we can arrive at an expected return for an option. Nevertheless, we can torture the animation a bit and make it calculate the average or expected return for a stock forecast.

What we want to do is create an artificial call option that has the same probability distribution as the underlying stock. To create that probability distribution, we set the strike price of the call option equal to zero and set the option price equal to the trading price of the stock.

When we find the average return for this imaginary option's probability distribution, that is also the expected return for the stock forecast.

1. Click three times on Clear.
2. Click twice on Hide.
3. Click on Simulate Price Change.
4. For Current Asset Price, enter 100.
5. Click on Draw Your Forecast.
6. Click on Draw Option Forecast.
7. Click on Call.
8. For Days to Expire, enter 365.
9. For Call Strike Price, enter 0.00
10. For Call Option Price, enter 100.00.
11. For Expected CC Return, enter 10.
12. For Standard Deviation, enter 30.0.
13. Click on Draw Your Forecast.

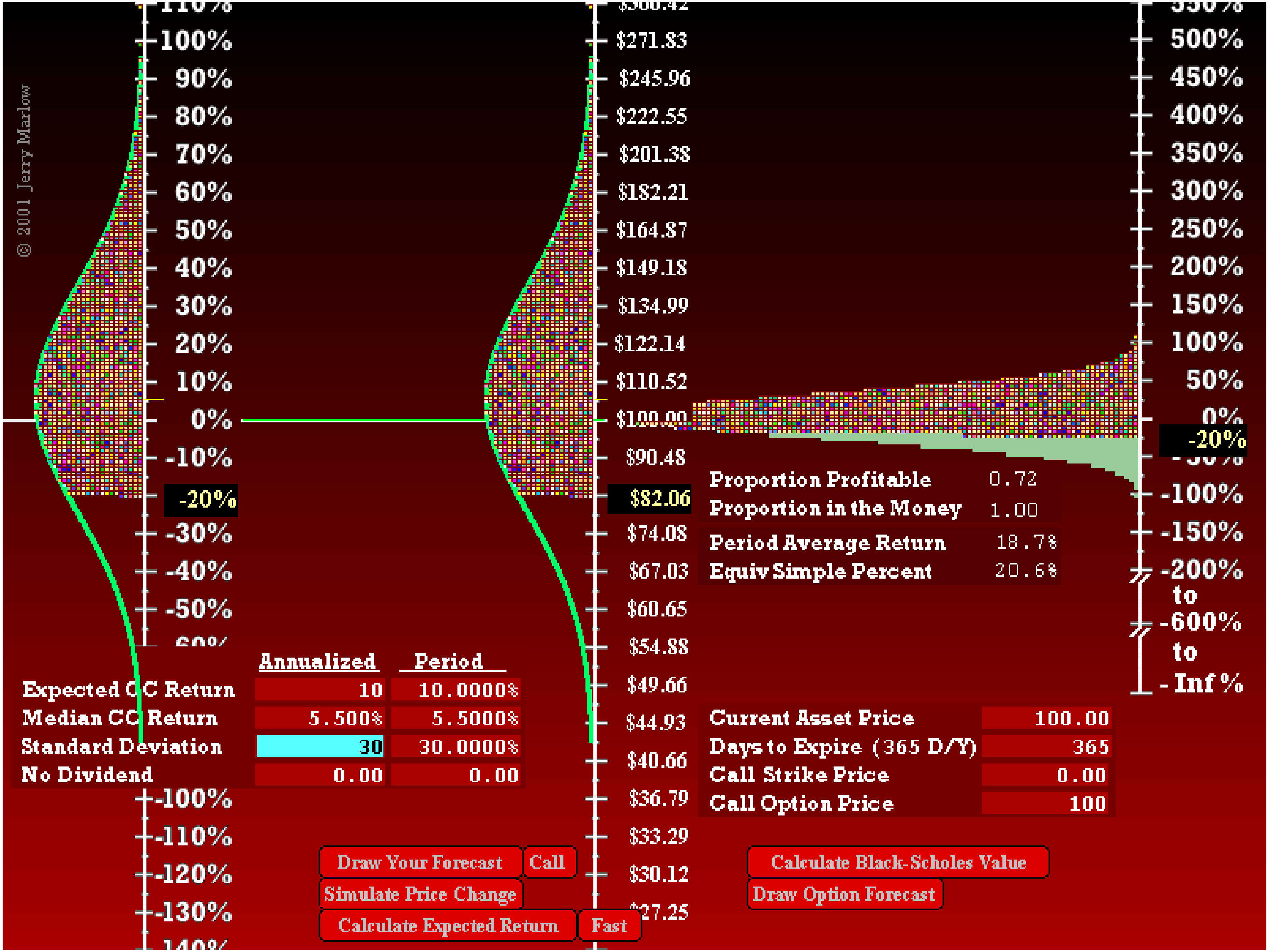
The animation first draws the return forecast for the underlying on the underlying-return axes. It then draws the same probability distribution on the option return axes.

The scale on the option return axes is quite different, but the probability distribution is the same. If you look at it, you will see that both probability distributions extend from 95.5% to -84.5%.

Now, let's verify that the probability distribution for the option is the same as the probability distribution for the underlying.

14. Click on Draw Option Forecast.

The probability distributions are the same.


© 2001 Jerry Marlow
100%
90%
80%
70%
60%
50%
40%
30%
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10%
0%
-10%
-20%
-30%
-40%
-50%
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$222.55
$201.38
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$164.87
$149.18
$134.99
$122.14
$110.52
$100.00
$90.48
$82.06
$74.08
$67.03
$60.65
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$44.93
$40.66
$36.79
$33.29
$30.12
500%
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400%
350%
300%
250%
200%
150%
100%
50%
0%
-20%
-100%
-150%
-200%
to
-600%
to
- Inf %
Proportion Profitable 0.72
Proportion in the Money 1.00
Period Average Return 18.7%
Equiv Simple Percent 20.6%
Annualized
Period
Expected CC Return 10 10.0000%
Median CC Return 5.500% 5.5000%
Standard Deviation 30 30.0000%
No Dividend 0.00 0.00
-100%
-110%
-120%
-130%
Current Asset Price 100.00
Days to Expire (365 D/Y) 365
Call Strike Price 0.00
Call Option Price 100
Draw Your Forecast
Call
Simulate Price Change
Calculate Expected Return
Fast
Calculate Black-Scholes Value
Draw Option Forecast

A stock's expected return is not the middle of its probability distribution. It's the middle plus half the standard deviation squared. With a high-volatility forecast, a stock's expected return may be positive, but its median return negative.

To calculate the "option's" expected return, the animation is going to sweep through the option's probability distribution and calculate a running average of all the returns in the probability distribution. You're running another structured Monte Carlo simulation.

When you see a return beyond the extremes of the probability distributions, remember that the outline probability distributions are drawn at the 97.7% confidence level. There is a 0.3% chance that an outcome will be above or below the probability distribution.

15. Click a few times on Calculate Expected Return.

Observe the results in the field labeled Period Average Return. This is the running average of the outcomes swept through thus far.

16. Click on Fast to the right of Calculate Expected Return.

The animation continues to calculate the average of the outcomes in the probability distribution.

17. To speed up the animation, click on Fastest.

The average of all the returns in the distribution is 10.0%. Hence, the expected return is 10.0%

When you're working with probability distributions in which geometric rates of return are normally distributed, the expected return can be found by a simple formula:

Expected CC return = median CC return + 0.5(standard deviation)^2

In our example,

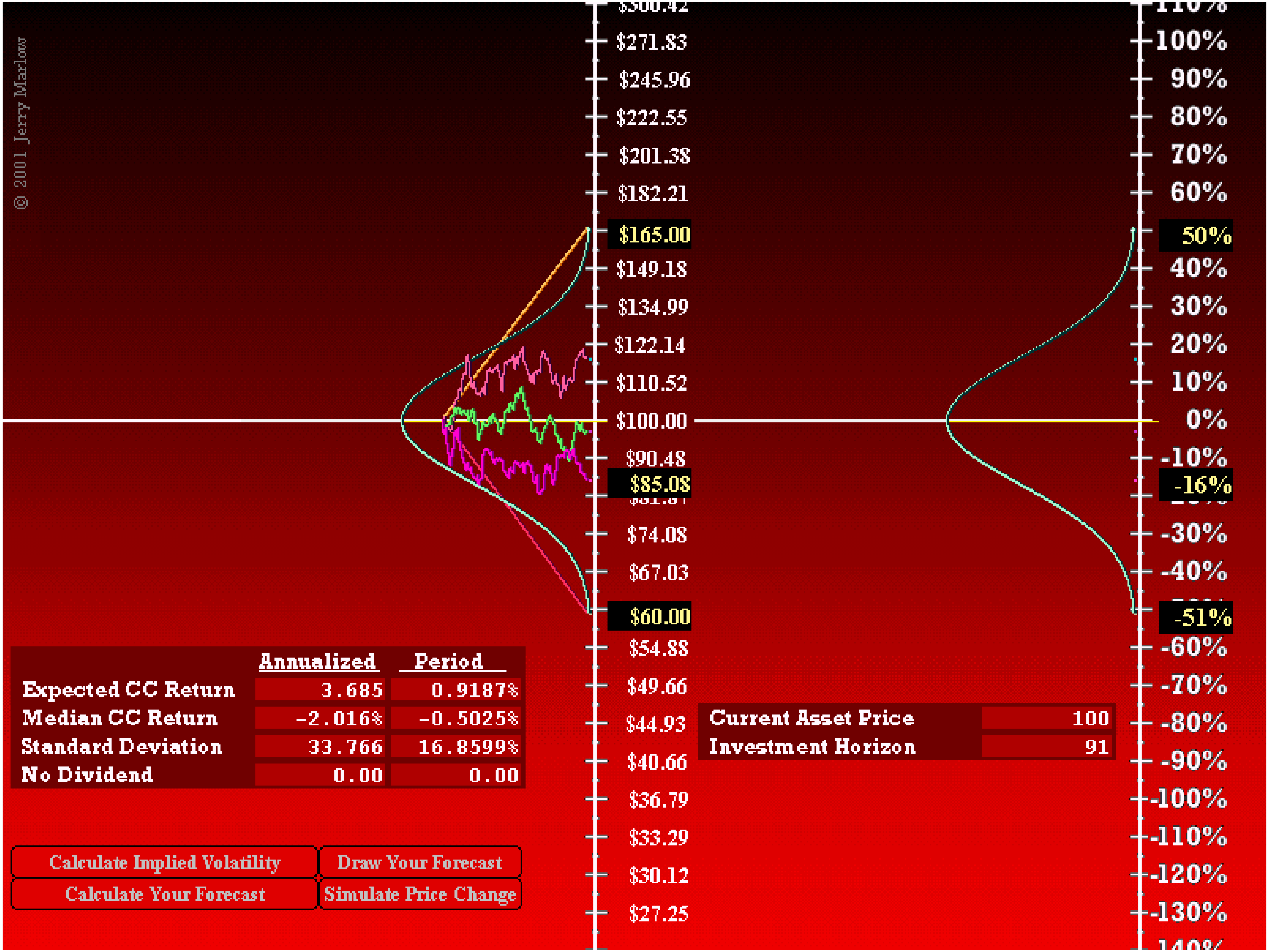
Expected CC Return = .0550 + 0.5(.30)^2
= .0550 + (0.5)(.0900)
= .0550 + .0450
= .1000
= 10%

When investment advisors give you a forecast for a stock, they usually quote an expected return and a standard deviation. Accordingly, the animation accepts the expected return as an input and calculates the forecast's median return.

The relationship among expected return, standard deviation, and median return means that, for stocks with high volatility forecasts, expected returns may be positive, but median returns negative. Consider a stock with an expected return of 6% and a standard deviation of 40%:

Median CC Return = Expected CC Return - 0.5 (Standard Deviation)^2
= 0.06 – (0.5)(.40)^2
= 0.06 – (0.5)(0.16)
= 0.06 - .08
= -.02
= -2.00%

In other words, a high-volatility stock may have a positive expected return, but chances are better than even that you'll lose money.



© 2001 Jerry Marlow
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-10%
-16%
-30%
-40%
-51%
-60%
-70%
-80%
-90%
-100%
-110%
-120%
-130%
Annualized
Period
Expected CC Return 3.685 0.9187%
Median CC Return -2.016% -0.5025%
Standard Deviation 33.766 16.8599%
No Dividend 0.00 0.00
Current Asset Price 100
Investment Horizon 91
Calculate Implied Volatility
Draw Your Forecast
Calculate Your Forecast
Simulate Price Change

Expected return varies with time

Let's look at how probability distributions change shape with the length of your investment horizon.

Let's say you're interested in a stock that is trading today at $100.00. It pays no dividends. You are 99.7% confident that, three months from now, the price will be somewhere between $60.00 and $165.00.

If we assume that the volatility of the stock's potential price path will not change and we extend this outlook to a year, what does the forecast look like?

First we calculate your three month forecast.

1. Click three times on Clear.
2. Click two or three times on Hide.
3. Click on Calculate Your Forecast.
4. For Current Asset Price, enter 100.00.
5. For Investment Horizon, enter 91.
6. For Highest possible price, enter 165.00.
7. For Lowest possible price, enter 60.00.
8. Click on Calculate Your Forecast.

The period expected return is .9187%. The period median continuously compounded rate of return is –0.5025. The period standard deviation is 16.8599.

Let's get a sense of what the volatility or uncertainty associated with this forecast looks like.

9. Click on Simulate Price Change a few times.

With this amount of volatility, in 91 days, the price path can deviate only so far from the median return.

When you calculate your forecast in this manner, from the period forecast, the animation calculates and displays the annualized forecast.

In our example, the annualized expected return is 3.685%.

The annualized median continuously compounded rate of return is -2.016%.

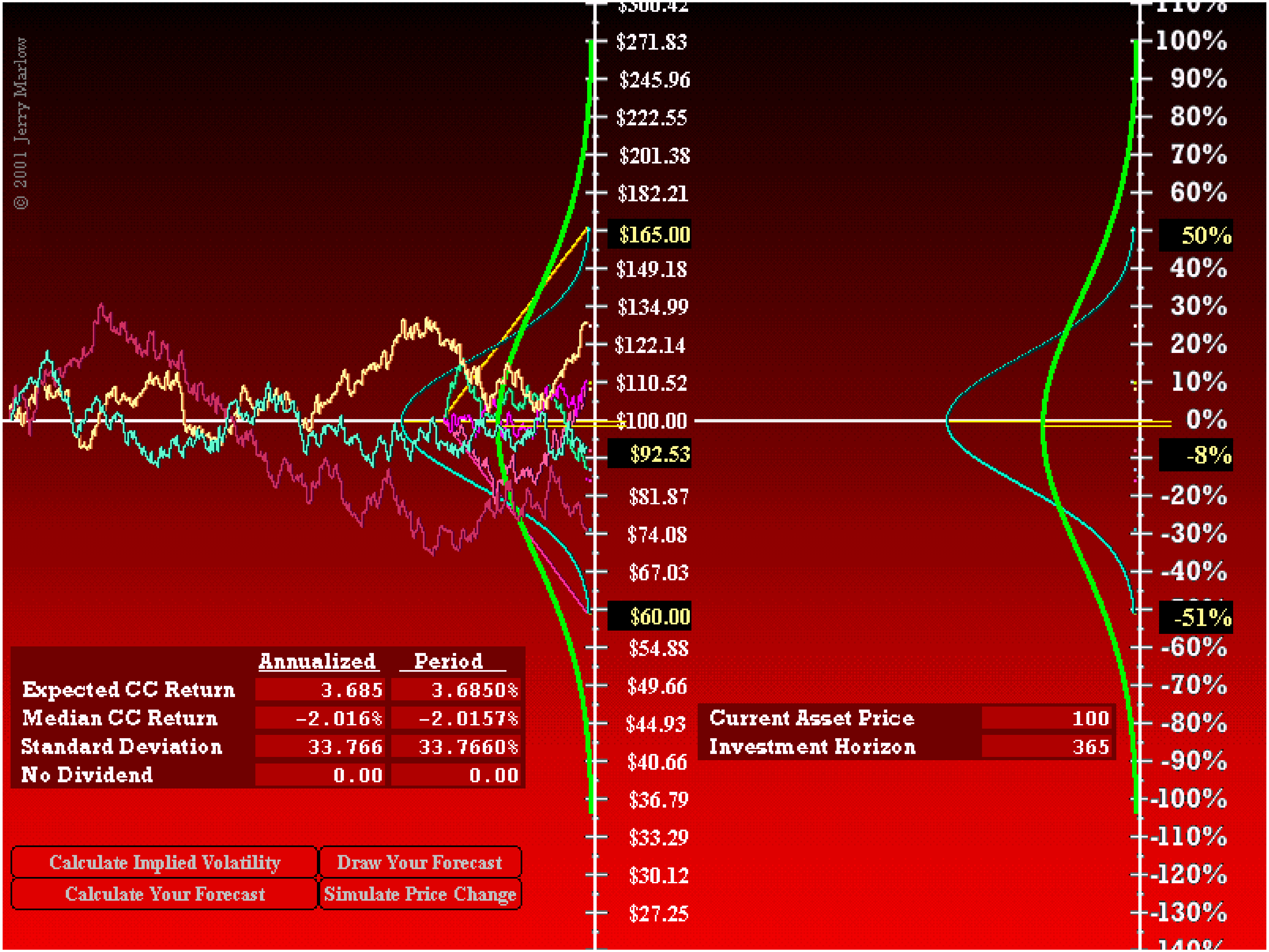
The annualized standard deviation is 33.766%.

Let's see what the annualized probability distribution looks like.

10. For Investment Horizon, enter 365.
11. Click on Draw Your Forecast.

Let's see what potential price paths for this forecast for one year look like.

12. Click on Simulate Price Change a few times.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100%
90%
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-8%
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-30%
-40%
-51%
-60%
-70%
-80%
-90%
-100%
-110%
-120%
-130%
Annualized
Period
Expected CC Return 3.685 3.6850%
Median CC Return -2.016% -2.0157%
Standard Deviation 33.766 33.7660%
No Dividend 0.00 0.00
Current Asset Price 100
Investment Horizon 365
Calculate Implied Volatility
Draw Your Forecast
Calculate Your Forecast
Simulate Price Change

Uncertainty varies with the square root of time

The day-to-day volatility is the same as it was for the 91-day period. Letting the simulation run for a year means that during that amount of time, the price path can deviate farther from the distribution's median return.

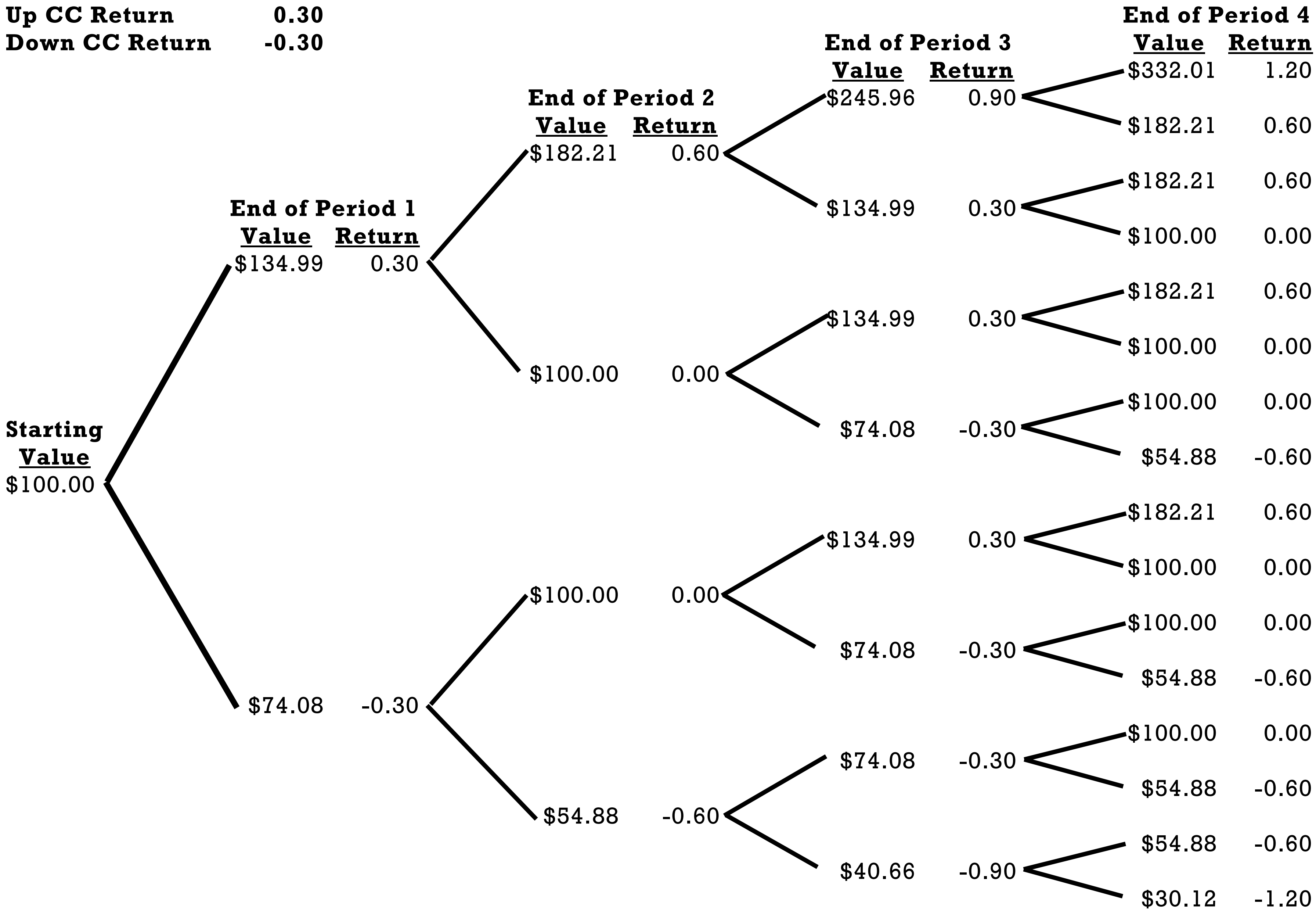
When you use geometric Brownian motion to model rates of return, the rules that govern how the expected return and standard deviation change over time are these:

The expected return varies with time. For example, if the expected return for one quarter is 3%, then the expected return for one year (four quarters) is 12%.

$$\begin{aligned}\text{Expected CC return} &= (3\%)(4)\\ &= 12\%\end{aligned}$$

The standard deviation or uncertainty varies with the square root of time. For example, quadrupling the investment horizon doubles the uncertainty of the end-of-period price. If the standard deviation for one quarter is 30%, the standard deviation for one year (four quarters) is 60%.

$$\begin{aligned}\text{Standard deviation} &= (30\%)(\sqrt{4})\\ &= (30\%)(2)\\ &= 60\%\end{aligned}$$



Standard Deviation*	0.30	0.42426	0.51961	0.60
	$(\sqrt{1})(.030) = 0.30$	$(\sqrt{2})(0.30) = 0.42426$	$(\sqrt{3})(0.30) = 0.51961$	$(\sqrt{4})(0.30) = 0.60$

Why does volatility vary not with time but with the square root of time? Because once prices get away from the median return, at every fork in the price-path tree, half the possible price paths lead back toward the median return.

If expected return varies with time, why, you might ask, does uncertainty or volatility vary with the square root of time?

To get a sense of why volatility varies with the square root of time, we can construct a simpler tree of possible price paths. We can take a look at the pattern of how end-of-period prices and returns differ from median prices and returns.

In the price-path tree above, we have very simple rules: At every fork in the tree, a price either goes up by a continuously compounded rate of 30% or it goes down by -30%. Up and down price movements are equally likely. The median return is 0.

You can begin at the starting value of $100.00 on the left and follow possible price paths across to the right. At the end of each period, you see the end-of-period value and the end-of-period cumulative return.

You'll notice several things: If you follow the extreme path at the top, you'll notice that the cumulative return increases in proportion to time. At the end of period 1, the cumulative return is 0.30 or 30%; at the end of period 2, it's 0.60; at the end of period 3, it's 0.90; at the end of period 4, it's 1.20.

If you follow the extreme path at the bottom, you see a similar pattern. The cumulative period return varies with time: -0.30, -0.60, -0.90, -1.20.

If the extreme price paths were the only available price paths, then the standard deviation would vary with time.

Standard deviation, however, measures the average difference between returns and the median return. In between the extreme paths, many of the potential price paths head back toward the median return. Those return outcomes differ from the median much less than do the returns on the outer extremes.

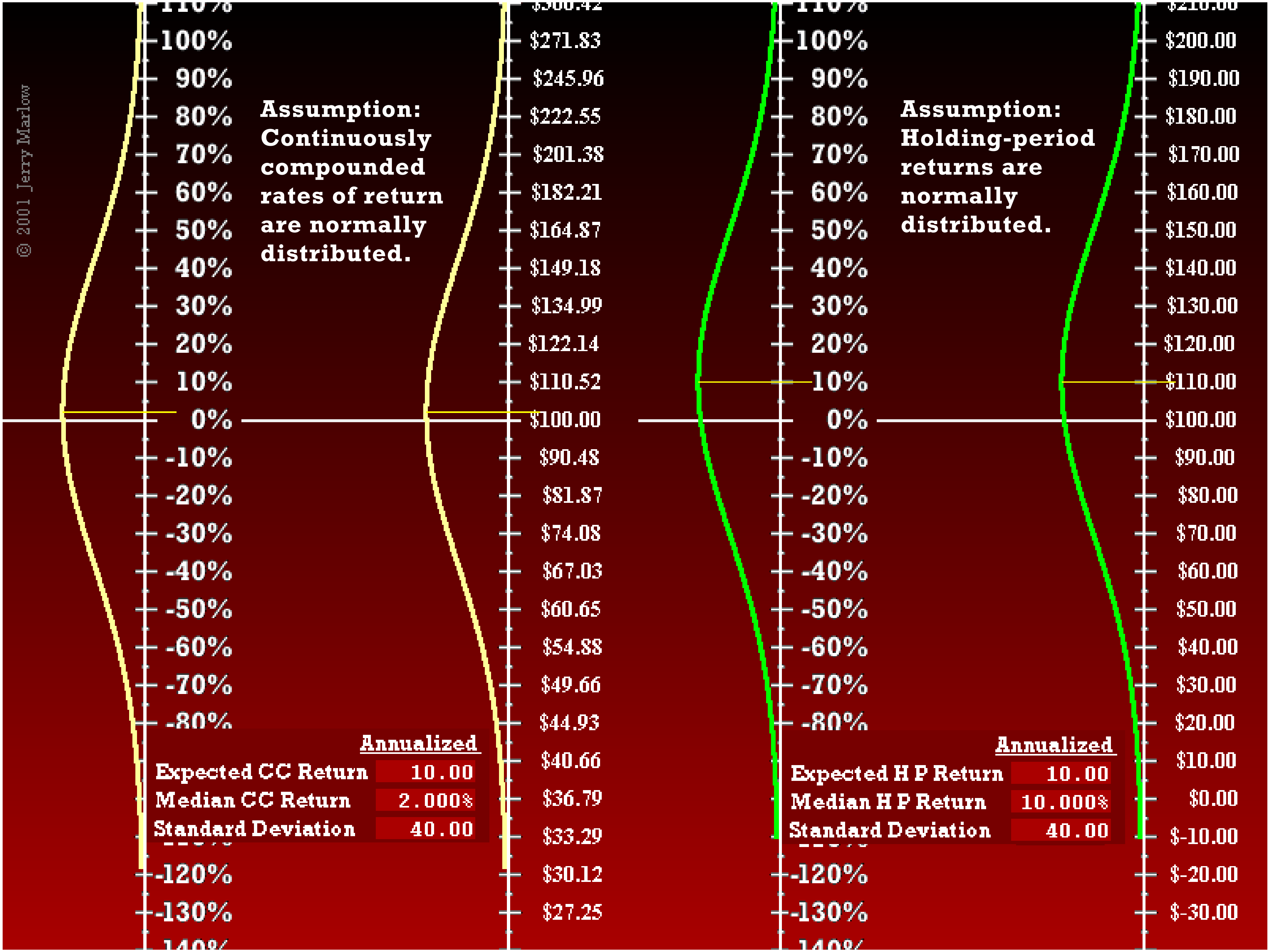
At the bottom of the chart, we've calculated the standard deviation of the cumulative returns at the end of each period.

To show what it means to say that standard deviation varies with the square root of time, we show that the period-2 standard deviation is equal to the period-1 standard deviation multiplied by the square root of 2. The period-3 standard deviation is equal to the period-1 standard deviation multiplied by the square root of 3. The period-4 standard deviation is equal to the period-1 standard deviation multiplied by the square root of 4.

A geometric-Brownian-motion price-path tree has many more potential price paths than the tree above, but the volatility pattern is essentially the same. Once prices get away from the median return, half the potential price paths lead back toward the median.

---

*To calculate the standard deviation of a sample, you divide the sum of the squares of the differences by the number of observations minus one. To calculate the standard deviation of an entire population, you divide by the number of observations. In the chart above, we are calculating standard deviations of entire populations. Accordingly we divide sums of squares of differences by the number of observations. In Excel, you would use the function STDEVP.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Assumption: Continuously compounded rates of return are normally distributed.
$271.83
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$67.03
$60.65
$54.88
$49.66
$44.93
$40.66
$36.79
$33.29
$30.12
$27.25
Annualized
Expected CC Return 10.00
Median CC Return 2.000%
Standard Deviation 40.00
Assumption: Holding-period returns are normally distributed.
$200.00
$190.00
$180.00
$170.00
$160.00
$150.00
$140.00
$130.00
$120.00
$110.00
$100.00
$90.00
$80.00
$70.00
$60.00
$50.00
$40.00
$30.00
$20.00
$10.00
$0.00
$-10.00
$-20.00
$-30.00
Annualized
Expected H P Return 10.00
Median H P Return 10.000%
Standard Deviation 40.00

When your portfolio manager or broker shows you a distribution of returns, ask whether he or she is talking continuously compounded or holding-period returns

Oddly enough, portfolio-management theory ordinarily does not assume that stock returns expressed as geometric or continuously compounded rates of return are normally distributed. Rather portfolio-management theory usually assumes that annual, holding-period returns are normally distributed.

What's the difference?

Let's say you're interested in a stock that, today, is trading at $100. Someone tells you that the stock has an expected return of 10% and a standard deviation of 40%. Depending on whether he or she is thinking and talking geometric or holding-period returns, the distribution of end-of-year prices being predicted is very different.

The return and price forecasts above left show forecasts under the assumption that continuously compounded rates of return are normally distributed.

The return and price forecasts above right show forecasts under the assumption that holding-period returns are normally distributed. The table below highlights some of the differences.

	If geometric rate of return	**If holding-period return**
Maximum end-of-year value at 99.7% confidence level (+3 standard deviations)	$338.72	$230.00
Average end-of-year value	$110.52	$110.00
Median end-of-year value	$102.02	$110.00
Minimum end-of-year value at 99.7% confidence level (-3 standard deviations)	$30.72	-$10.00

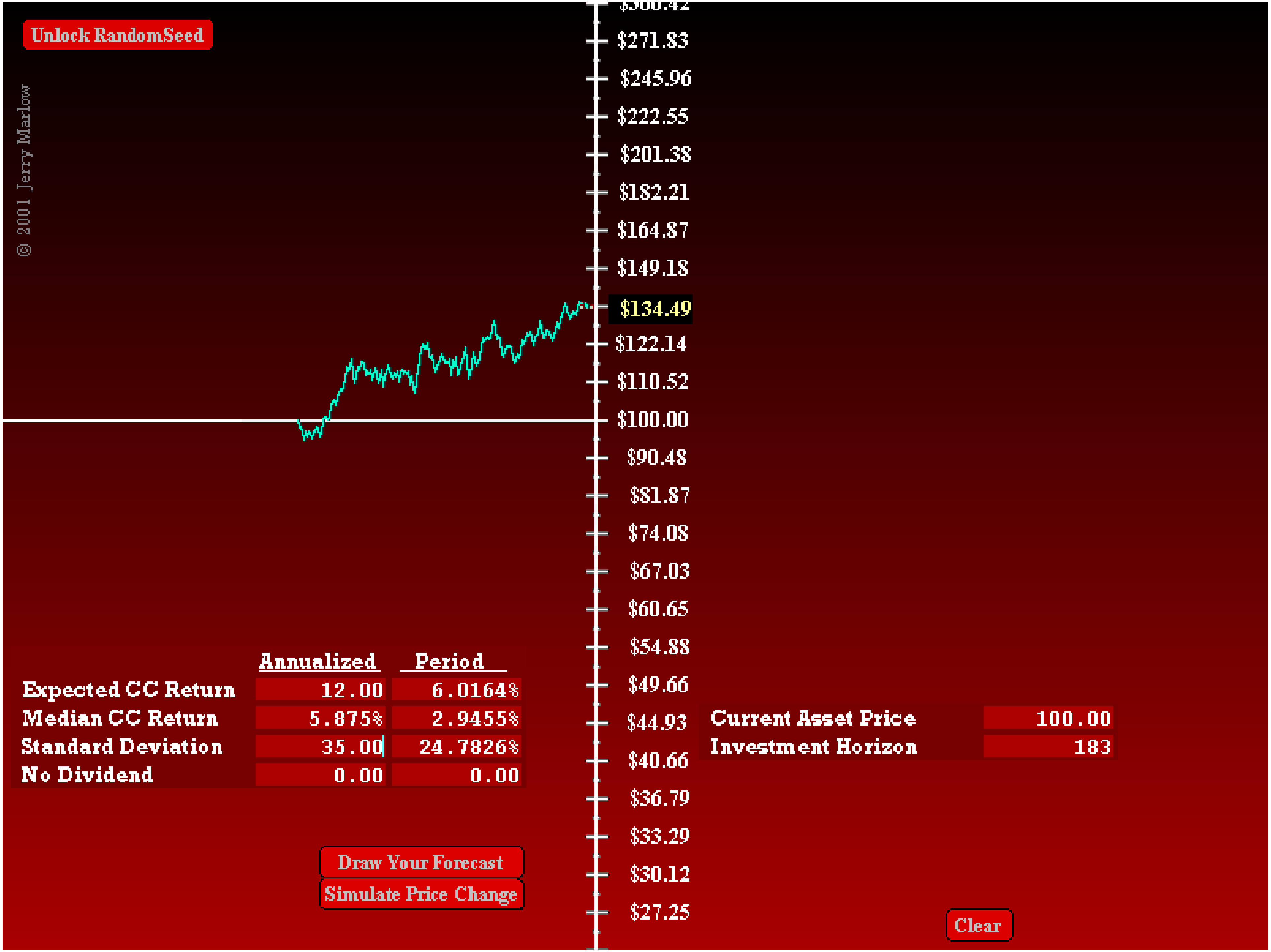
How is an end-of-year price of -$10 possible? Ask someone who believes that holding-period returns are normally distributed.

When your portfolio manager or broker shows you a normal distribution of returns or gives you a forecast, clarify whether he or she is talking continuously compounded rates of return or holding-period returns. Be prepared for the possibility that your portfolio manager or broker may not know or—Heaven forbid!— may not know the difference.

If they do not know the difference, insist that they buy a copy of *Black-Scholes Made Easy* for everyone in their firm.

(Your animation is not programmed to draw any graphs in which holding-period returns are normally distributed.)

How dividends affect price paths and forecasts



Unlock RandomSeed
© 2001 Jerry Marlow
$271.83
$245.96
$222.55
$201.38
$182.21
$164.87
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$122.14
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$67.03
$60.65
$54.88
$49.66
$44.93
$40.66
$36.79
$33.29
$30.12
$27.25
Annualized
Period
Expected CC Return 12.00 6.0164%
Median CC Return 5.875% 2.9455%
Standard Deviation 35.00 24.7826%
No Dividend 0.00 0.00
Current Asset Price 100.00
Investment Horizon 183
Draw Your Forecast
Simulate Price Change
Clear

Lock Random Seed lets you create the same price path with variations

To see the effects that dividend payments and yields have on stock price paths and forecasts, we'll use the animation's Lock-Random-Seed capability. Lock Random Seed is kind of like the brake on a rigged roulette wheel. It causes Simulate Price Change to keep generating the same sequence of geometric-Brownian-motion perturbations and, hence—if nothing else is changed—the same price path over and over again.

Let's see what that looks like.

1. Click three times on Clear.
2. Click twice on Hide.
3. Click on Simulate Price Change.
4. For Current Asset Price, enter 100.
5. For Investment Horizon, enter 183.
6. For Expected CC Return, enter 12.
7. For Standard Deviation, enter 35.
8. Click on Simulate Price Change.
9. To make the Lock-Random-Seed command button visible, mouse around at the top left of the screen.
10. Click on Lock Random Seed.

Watch the existing price path closely.

11. Click on Simulate Price Change

Notice that the animation traces the same simulated price path again.

You can keep clicking on Simulate Price Change again and again and you'll get the same price path every time.

When you want to go back to random price paths, click on Unlock Random Seed.

How does Lock Random Seed work?

To simulate Brownian-motion price paths, the animation generates a series of random numbers, finds the number in the stock's one-day probability distribution that corresponds to that random number, and increases or decreases the stock's price accordingly.

To generate random numbers, a computer starts with a number called the random seed and performs a sequence of operations on it. Whenever a computer starts with the same number as its random seed, it generates the same sequence of random numbers.

Lock Random Seed causes the computer to start with the same random seed next time that it used last time. It generates the same sequence of random numbers which produces the same price path or—if you've changed anything—the same sequence of Brownian-motion jolts to the price.

12. To see what the same sequence of perturbations look like with a different volatility, for Standard Deviation, enter 55.00.
13. Click on Simulate Price Change.

The so-called random numbers that computers generate aren't really random. Purists call them pseudo random numbers. Numbers that are really random are trickier to generate.

14. Click on Unlock Random Seed.

(If ever the animation seems to be doing bizarre things, check to see if the random seed is locked.)

"Anyone who considers arithmetic methods of producing random digits is, of course, in a state of sin."

John von Neumann


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100%
90%
80%
70%
60%
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40%
30%
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10%
6%
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-10%
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-30%
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-50%
$271.83
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$222.55
$201.38
$182.21
$164.87
$149.18
$134.99
$122.14
$110.52
$100.00
$90.48
$85.43
$74.08
$67.03
$60.65
$54.88
$49.66
$44.93
$40.66
$36.79
$33.29
$30.12
$27.25
Days until ex-dividend
Dividend amount
First dividend 45 10.00
Second dividend 90 10.00
Third dividend
Fourth dividend
Fifth dividend
Sixth dividend
Seventh dividend
Eighth dividend
Calculate Net Present Value
Using Risk-free Rate
Using Expected Return
Dividend NPV 19.78 19.56
Close Dividend Schedule
To calculate dividend net present value, the model discounts the stream of dividends expected to be paid during the investment period.
The model calculates two different dividend net present values. For one, the model uses as the discount rate the risk-free rate of return. For the other, it uses the expected return.
Annualized
Period
Expected CC Return 12.00 6.0164%
Median CC Return 12.000% 6.0164%
Standard Deviation 0.00 0.0000%
Dividend NPV 19.56
CC Risk-free Rate 6.00 3.0082%
Current Asset Price 100.00
Investment Horizon 183
-110%
-120%
-130%
Draw Your Forecast
Simulate Price Change
Display Div Schedule
No Div

You can expect a dividend payment to reduce the price of a stock by the amount of the dividend

Thus far, we have been using as our examples stocks that pay no dividends.

15. Click once on Clear.
16. Click twice on Draw Your Forecast.

When a stock pays no dividends, price appreciation accounts for all of the stock's return. Consequently, the return probability distribution and the end-of-period price probability distribution sit at the same height.

17. Click on Simulate Price Change.

When a stock pays no dividends, in the model, geometric Brownian motion accounts for all of the price changes.

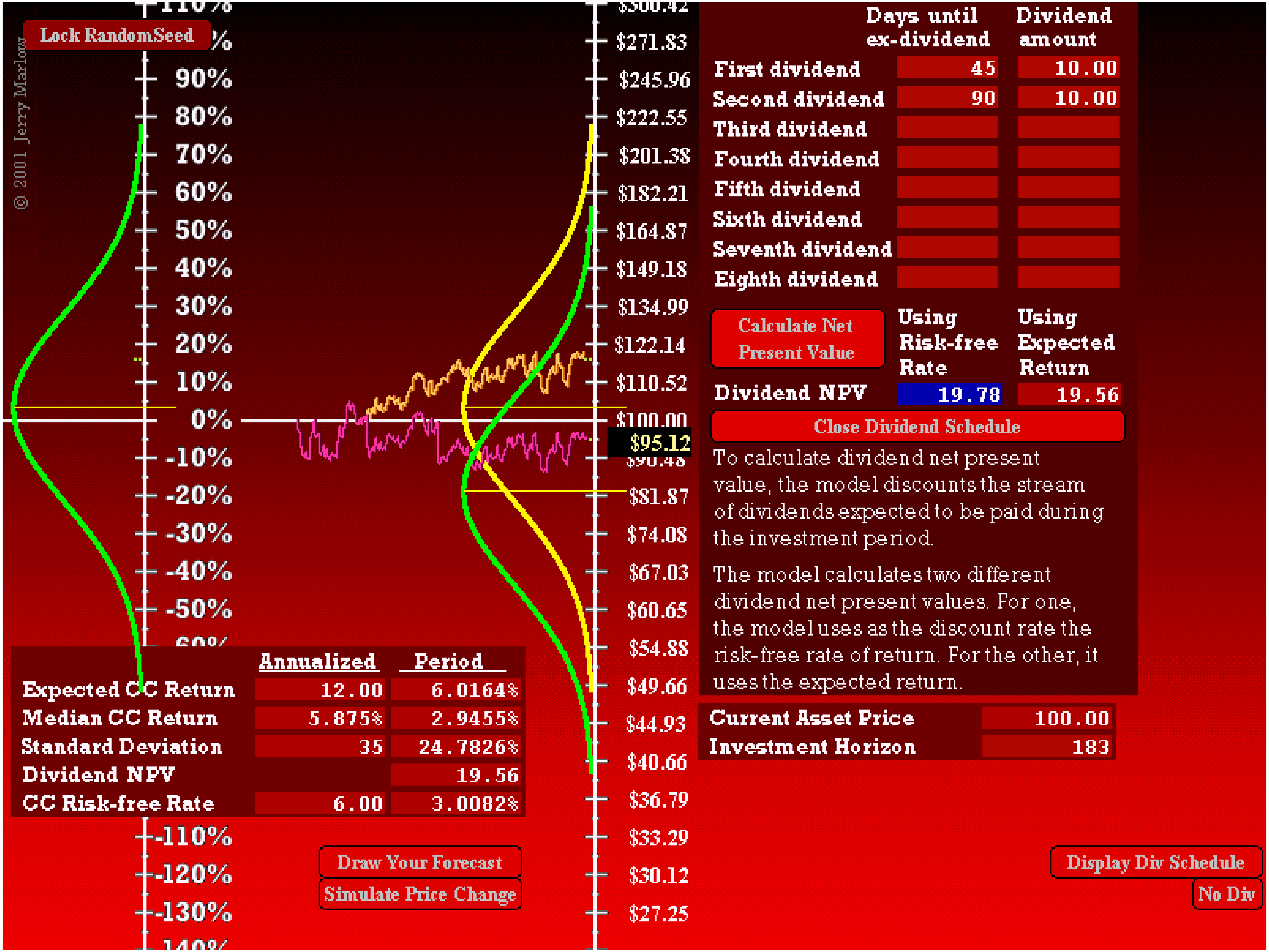
When a stock pays dividends, in theory, the stock's price is reduced on the ex-dividend date by the amount of the dividend. Accordingly, on the ex-dividend date the model reduces the price of the stock by the amount of the dividend.

To see in a stark way the effect that dividend payments have on a potential price path, we will remove all uncertainty from the forecast, generate a price path, introduce dividends, then generate a price path that shows the price drops that accompany dividend payments.

18. For Standard Deviation, enter 0.00.
19. Click on Simulate Price Change.
20. Click on Display Dividend Schedule.
21. For CC Risk-free rate, enter 6.00.
22. For the first dividend, under Days until ex-dividend, enter 45.
23. Under Dividend amount, enter an absurdly high dividend: 10.00.
24. For the second dividend, under Days until ex-dividend, enter 90.
25. Under Dividend amount, enter 10.00.
26. Click on Calculate Net Present Value.
27. Click on Simulate Price Change.

At days 45 and 90, you see the price drop by $10.00 because of the dividend payments.

Now we will look at the effect that dividend payments have on a volatile price path.



Lock RandomSeed
© 2001 Jerry Marlow
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$40.66
$36.79
$33.29
$30.12
$27.25
Days until ex-dividend
Dividend amount
First dividend 45 10.00
Second dividend 90 10.00
Third dividend
Fourth dividend
Fifth dividend
Sixth dividend
Seventh dividend
Eighth dividend
Calculate Net Present Value
Using Risk-free Rate
Using Expected Return
Dividend NPV 19.78 19.56
Close Dividend Schedule
To calculate dividend net present value, the model discounts the stream of dividends expected to be paid during the investment period.
The model calculates two different dividend net present values. For one, the model uses as the discount rate the risk-free rate of return. For the other, it uses the expected return.
Annualized
Period
Expected CC Return 12.00 6.0164%
Median CC Return 5.875% 2.9455%
Standard Deviation 35 24.7826%
Dividend NPV 19.56
CC Risk-free Rate 6.00 3.0082%
Current Asset Price 100.00
Investment Horizon 183
Draw Your Forecast
Simulate Price Change
Display Div Schedule
No Div

Dividends shift the price probability distribution below the return probability distribution

To see the effect, we will use the animation's Lock Random Seed capability.

28. For Standard Deviation, enter 35.0.
29. Click on Clear.
30. Click on No Div.
31. Click on Draw Your Forecast.
32. Click on Simulate Price Change.
33. Click on Lock Random Seed.
34. Click on Simulate Price Change again.

Notice that you get the same simulated price path.

Now we'll see what that price path looks like with dividend payments.

35. Click on Display Dividend Schedule.
36. For the first dividend, under Days until ex-dividend, enter 45.
37. Under Dividend amount, enter 10.00.
38. For the second dividend, under Days until ex-dividend, enter 90.
39. Under Dividend amount, enter 10.00.
40. Click on Calculate Net Present Value.
41. Click on Simulate Price Change.

You see that, on each ex-dividend day, the dividend payment lowers the price of the stock. The end-of-period price is lower. Similarly, dividend payments lower the entire probability distribution of the price forecast.

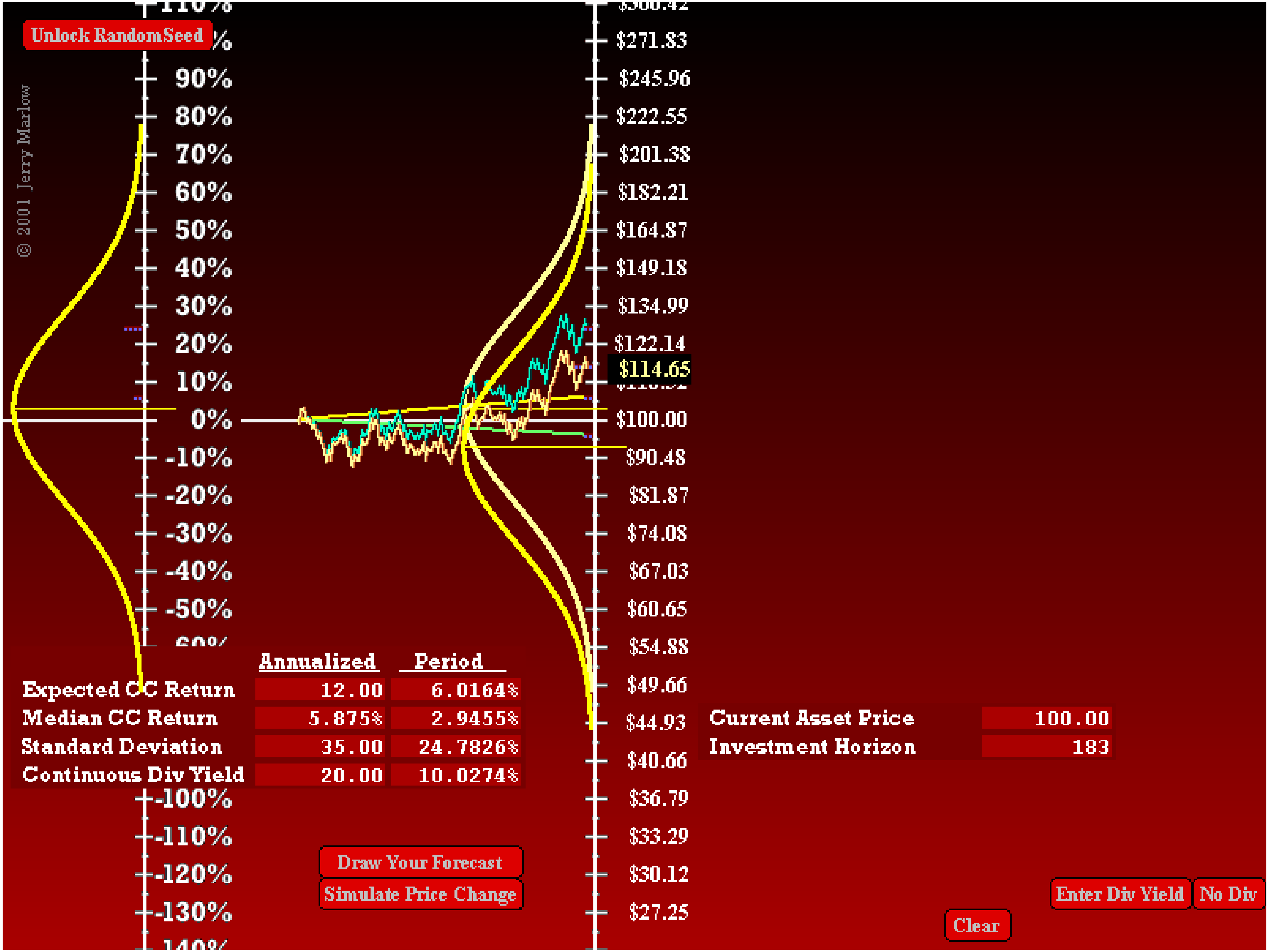
42. Click on Unlock Random Seed.
43. Click on Draw Your Forecast.
44. Click on Simulate Price Change a few more times.
45. Click on Simulate Price Change Fastest.
46. Click on Draw Your Forecast.

Dividend payments shift the price forecast lower than the return forecast. This shift makes sense because return is defined as price appreciation plus dividend income.

If you plan to invest in options, the difference between the return forecast and the price forecast is especially significant. Option payoffs are based on price performance, not return.

Later, when we look at circumstances under which it may be advantageous to exercise an option early, you will see that one of the most important considerations is the attempt to capture value from dividend payments.

To work with dividends, the animation requires that you first enter the investment horizon or time to expiration. If the time to expiration is blank or zero, the animation throws you an alert.


Unlock RandomSeed
© 2001 Jerry Marlow
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$49.66
$44.93
$40.66
$36.79
$33.29
$30.12
$27.25
Annalized Period
Expected CC Return 12.00 6.0164%
Median CC Return 5.875% 2.9455%
Standard Deviation 35.00 24.7826%
Continuous Div Yield 20.00 10.0274%
Current Asset Price 100.00
Investment Horizon 183
-100%
-110%
-120%
-130%
Draw Your Forecast
Simulate Price Change
Enter Div Yield
No Div
Clear

A dividend yield shifts the price probability distribution below the return probability distribution

In addition to buying options on stocks, you also can buy options on market indices. Instead of being modeled as paying a schedule of cash dividends, indices often are modeled as paying a dividend yield.

You can enter a dividend yield into the animation. The effects are similar to those of entering a dividend schedule. A dividend yield shifts the price probability distribution below the return probability distribution.

First we'll look at the effect of a dividend yield on a "stock" that has zero volatility.

47. Click on Clear.
48. Click on No Div.
49. For Standard Deviation, enter 0.00.
50. Click on Simulate Price Change.
51. Click on Enter Div Yield.
52. For Continuous Dividend Yield, enter 20.00.
53. Click on Simulate Price Change.

You see that both price paths are straight lines, but the path that pays the dividend yield diverges below the price path that pays no dividend yield.

Now we'll look at the effect of a dividend yield on volatile price paths and price forecasts.

54. Click on No Div.
55. For Standard Deviation, enter 35.00.
56. Click on Draw Your Forecast.
57. Click on Simulate Price Change.

You see that the return forecast and the price forecast are at the same height.

58. Click on Lock Random Seed at the top left of the screen.
59. Click on Enter Div Yield.
60. For Continuous Dividend Yield, enter 20.00.
61. Click on Simulate Price Change.

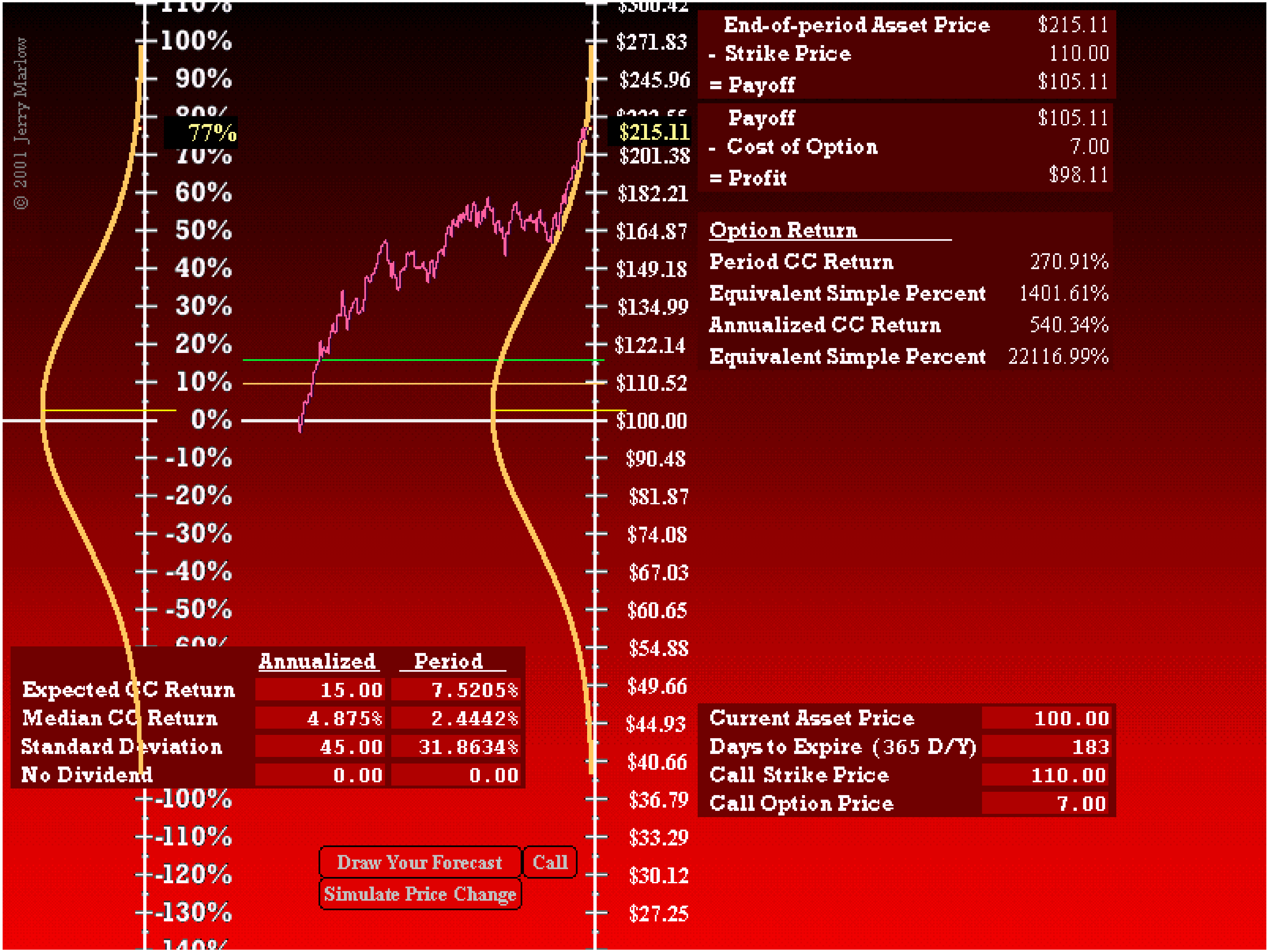
Notice that the volatile price path that pays a dividend yield diverges below the price path that pays no dividend yield.

62. Click on Unlock Random Seed.
63. Click on Draw Your Forecast..

Notice that the dividend yield shifts the price forecast below the return forecast.

If the stocks in an index pay a dividend yield, be sure to factor the yield into your calculations. Dividend yields affect the price paths upon which option payoffs are based.

Option Outcomes, Probability Distributions, and Expected Returns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77%
$215.11
End-of-period Asset Price $215.11
- Strike Price 110.00
= Payoff $105.11
Payoff $105.11
- Cost of Option 7.00
= Profit $98.11
Option Return
Period CC Return 270.91%
Equivalent Simple Percent 1401.61%
Annualized CC Return 540.34%
Equivalent Simple Percent 22116.99%
Annualized Period
Expected CC Return 15.00 7.5205%
Median CC Return 4.875% 2.4442%
Standard Deviation 45.00 31.8634%
No Dividend 0.00 0.00
Current Asset Price 100.00
Days to Expire (365 D/Y) 183
Call Strike Price 110.00
Call Option Price 7.00
Draw Your Forecast
Call
Simulate Price Change
100%
90%
70%
60%
50%
40%
30%
20%
10%
0%
-10%
-20%
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$49.66
$44.93
$40.66
$36.79
$33.29
$30.12
$27.25

A call option gives you the right to buy a stock at a pre-set price

When you buy a call option, you buy the right to buy a stock at a pre-set price called the strike price. With American-style options, you can exercise this right at any time up until the option's expiration date. With European-style options, you can exercise the option only on the expiration date—that is, at maturity.

If the price of the stock is above the strike price or goes above the strike price and you exercise the option, you receive the difference between the stock price and the strike price at the time of exercise.

Given a stock forecast and an option, the animation simulates potential price paths for the stock and shows the option's corresponding end-of-period payoffs.

1. Click on No Div.
2. Click three times on Clear.
3. Click two or three times on Hide.
4. Click on Simulate Price Change.
5. For Expected CC Return, enter 15.
6. For Standard Deviation, enter 45.
7. For Current Asset Price, enter 100.00.
8. Click on Call.
9. For Days to Expire, enter 183.
10. For Call Strike Price, enter 110.00.
11. For Call Option Price, enter 7.00.

The yellow line on the price axis represents the option's strike price. For a call option to be "in the money," the stock price must be above the yellow line.

The distance from the yellow line to the green line represents the option price or premium. For a call option to produce a profit, at the time of exercise the stock price must be above the green line.

12. Click twice on Draw Your Forecast.

The area of the probability distribution above the yellow line represents the probability that the option will finish in the money.

The area of the probability distribution above the green line represents the probability that the option will produce a profit.

13. Click on Simulate Price Change.

If the end-of-period price is below the strike price, you see that the payoff is zero, your "profit" is negative, and you lose all the money you invested in the option. You have a period continuously compounded rate of return of negative infinity and a holding-period return of -100%.

If the end-of-period price is above the strike price, you see that the payoff is equal to the end-of-period price minus the strike price.

14. Click on Simulate Price Change until you get a couple of winners.

Note the calculation of option payoffs for each simulated price path.


© 2001 Jerry Marlow
100%
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80%
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60%
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$67.03
$60.65
$54.88
$49.66
$44.93
$40.66
$36.79
$33.29
$30.12
$27.25
Portfolio Value 14000.79
Cash Allocation 9314.85
Risky Investment 4685.94
Elapsed Time (Years) 1.00
Annualized CC Return 33.56%
Equiv Simple Percent 39.88%
Annualized Period
Expected CC Return 15.00 7.5205%
Median CC Return 4.875% 2.4442%
Standard Deviation 45.00 31.8634%
No Dividend 0.00 0.00
CC Risk-free Rate 6.00 3.0082%
Current Asset Price 100.00
Days to Expire (365 D/Y) 183
Call Strike Price 110.00
Call Option Price 7.00
Draw Your Forecast
Call
Simulate Investment
Calculate Black-Scholes Value
Display Div Schedule
Analyze
Hide Simple Percent 42.802%
368.594%

Once you have your price forecast for a stock, you can simulate potential outcomes of investing in a call you hold until maturity

Based on your forecast and the option structure, you can simulate potential outcomes of investing in the option and holding it until expiration.

Let's say you have $10,000.00 in your portfolio. You want to invest $1,000.00 in the option.

15. Click on Show Simple Percent.
16. Click on Invest.
17. For CC Risk-free Rate, enter 6.0.
18. For Portfolio Value, enter 10000.00. (No $ or commas.)
19. For Risky Investment, enter 1000.
20. Click on Simulate Investment.

The simple percent box at the bottom right of the screen shows the equivalent simple percent of the continuously compounded rate of return on your investment in the option.

Regardless of how the option performs, the cash allocation earns the risk-free rate of return.

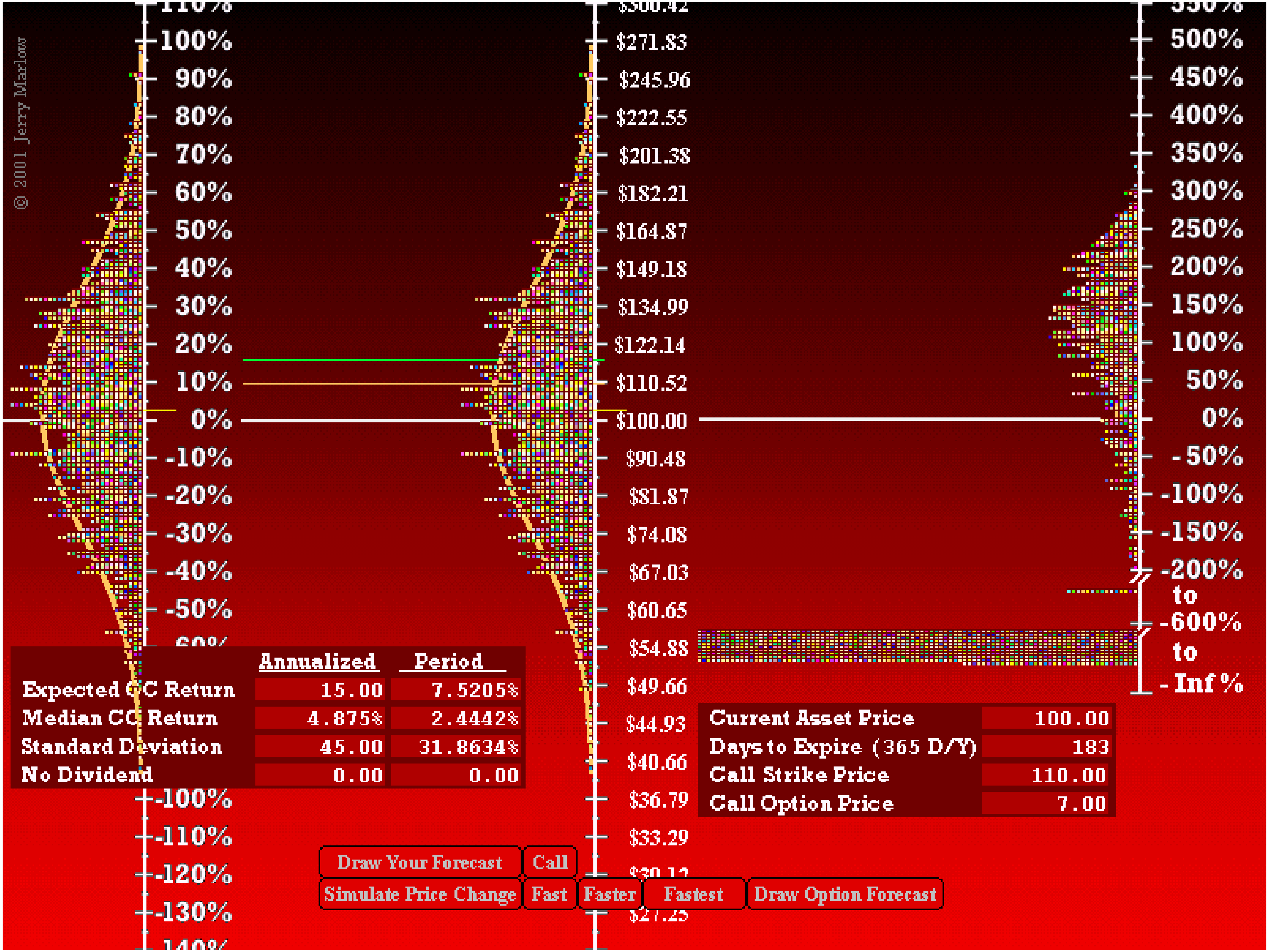
The animation displays the new value of the portfolio, the new value of the cash allocation, the proceeds of the investment in the option, the elapsed investment period in years, the annualized continuously compounded rate of return for the portfolio, and the equivalent simple percent.

To get a feel for how investing in call options can leverage your investment, make a few more investments of $1,000.00 in the option.

21. For Risky Investment, enter 1000.00.
22. Click on Simulate Investment.

If you wish to see only the effect on the portfolio of investing in the option, set the risk-free rate to zero.

23. For CC Risk-free Rate, enter 0.0.
24. For Portfolio Value, enter 10000.00. (No $ or commas.)
25. For Risky Investment, enter 1000.00.
26. Click on Simulate Investment.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200%
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100%
50%
0%
-50%
-100%
-150%
-200%
to
-600%
to
-Inf %
Annualized
Period
Expected CC Return 15.00 7.5205%
Median CC Return 4.875% 2.4442%
Standard Deviation 45.00 31.8634%
No Dividend 0.00 0.00
Current Asset Price 100.00
Days to Expire (365 D/Y) 183
Call Strike Price 110.00
Call Option Price 7.00
Draw Your Forecast
Call
Simulate Price Change
Fast
Faster
Fastest
Draw Option Forecast

A histogram of option outcomes approximates a probability distribution or return forecast for the option

Instead of displaying the option payoffs as numerical results, the animation can display option performance graphically as a return on your investment in the option.

27. To make the Invest data fields go away, click on Analyze.
28. To bring the option axes on screen, click once on Draw Option Forecast.
29. Click on Simulate Price Change.

To tabulate each option return, the animation adds a little square to the option axes.

You can simulate a price change and option return without drawing the price path.

30. Click on Fast immediately to the right of Simulate Price Change.

You can speed up the simulation.

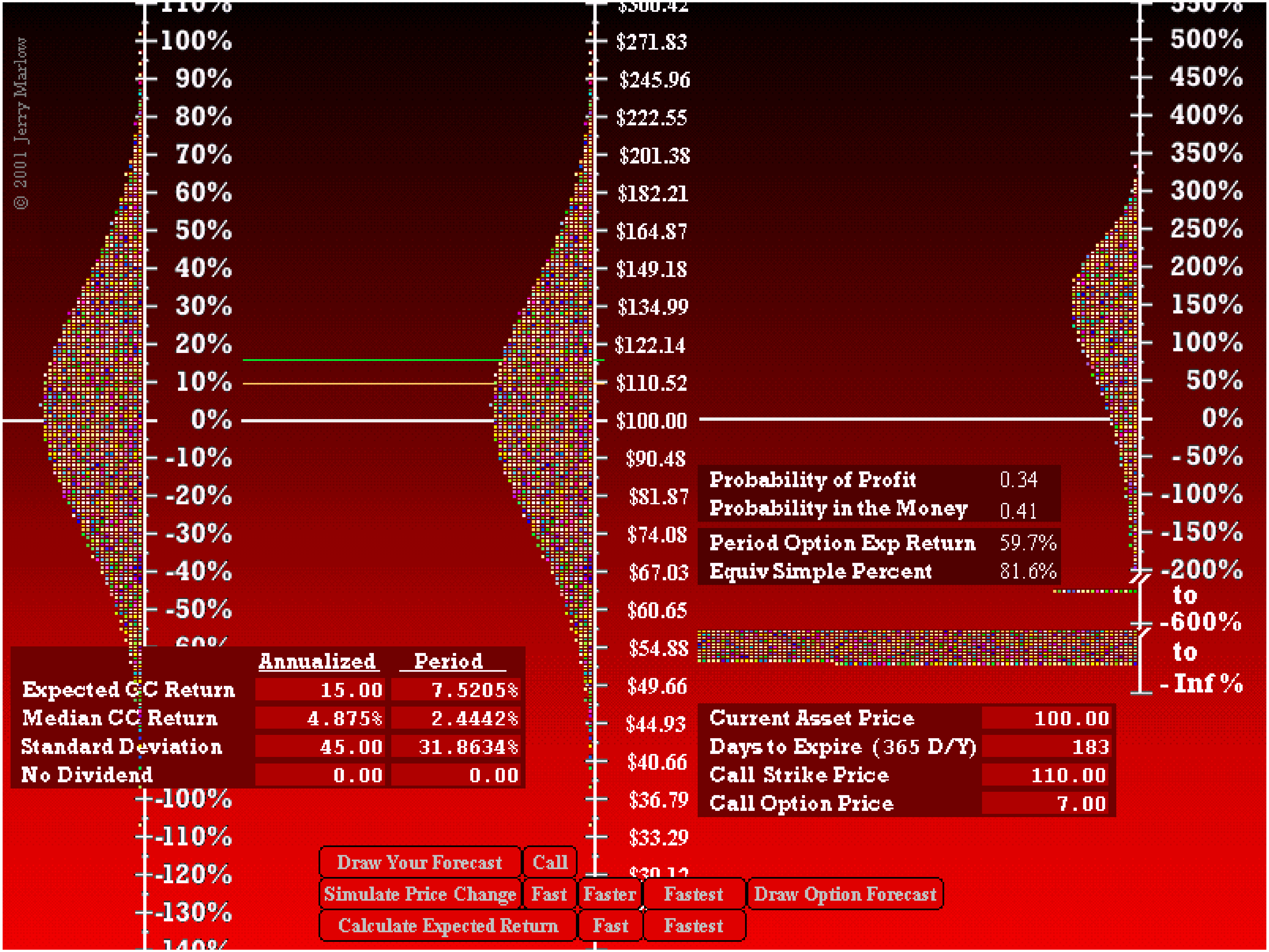
31. Click on Faster to the right of Simulate Price Change.

You can speed it up further.

32. Click on Fastest to the right of Simulate Price Change.

The histograms of squares on the left and in the middle of the screen approximates the probability distri-butions of the stock's return and price forecast.

Given this stock forecast and option structure, the histogram formed on the option axes approximates the probability distribution or forecast of the option return when the option is held to maturity. (For options, the words *expiration* and *maturity* mean the same thing.)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Annualized
Period
Expected CC Return 15.00 7.5205%
Median CC Return 4.875% 2.4442%
Standard Deviation 45.00 31.8634%
No Dividend 0.00 0.00
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to
-600%
to
- Inf %
Probability of Profit 0.34
Probability in the Money 0.41
Period Option Exp Return 59.7%
Equiv Simple Percent 81.6%
Current Asset Price 100.00
Days to Expire (365 D/Y) 183
Call Strike Price 110.00
Call Option Price 7.00
Draw Your Forecast
Call
Simulate Price Change
Fast
Faster
Fastest
Draw Option Forecast
Calculate Expected Return
Fast
Fastest

The expected return of an option held to maturity is the average of all the returns in its probability distribution

Because we can translate each end-of-period stock price into an option return, we can sweep down through the stock forecast and simultaneously sweep down through the option forecast. Along the way, we can calculate several properties of the option forecast:

- The option's probability of profit.
- The probability that the end-of-period stock price will be in the money.
- The period expected return of the option.
- The equivalent simple percent of the period option expected return.

We continue with the example that you've entered into the animation already.

33. Click once on Clear.
34. Click a few times on Calculate Expected Return.

We begin to sweep down through the forecasts. Initially—for this stock forecast and option structure—all the option outcomes are profitable and in the money. Accordingly, the proportion profitable and proportion in the money is 1.00 or 100%. When the option outcomes begin to fall below the green line and then the yellow line, the proportion profitable and proportion in the money will fall below 1.00.

For period average return, the animation is simply calculating a running average of the option returns thus far swept through.

35. Click on Fast immediately to the right of Calculate Expected Return.

Automatically, the animation continues to sweep through the stock forecast, display the corresponding option outcomes, and tabulate the proportion of outcomes profitable and in the money.

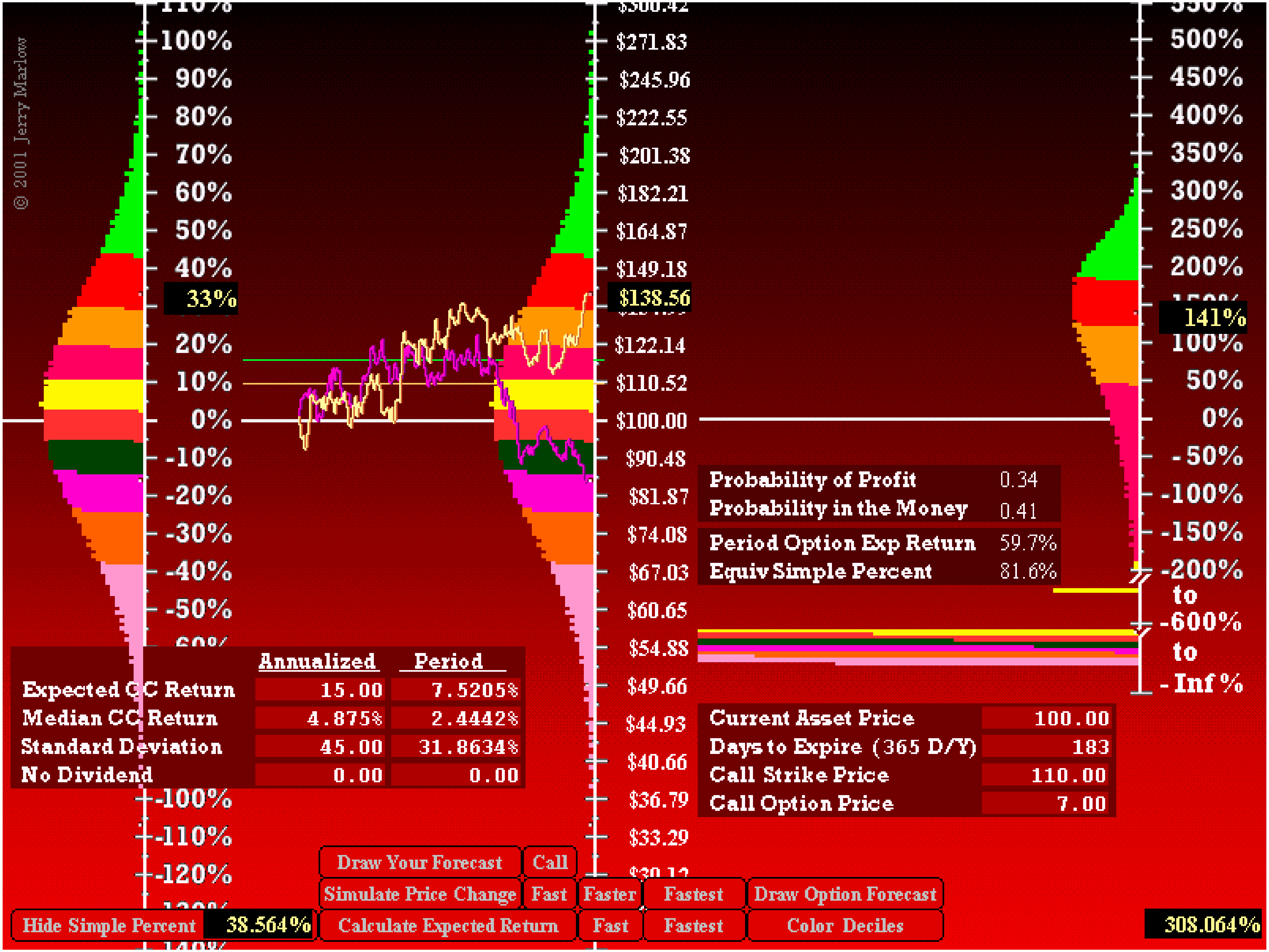
36. Click on Fastest to the right of Calculate Expected Return.

The animation more quickly sweeps through the probability distributions.

Once it finishes, the proportion of outcomes profitable becomes the probability of profit. The proportion of outcomes in the money becomes the probability of being in the money. The average return becomes the period expected return for the option.

When you click on Color Deciles, the animation uses the same techniques, performs the same calculations (though to greater precision), and divides each of the probability distributions into color-coded deciles.

37. Click on Color Deciles.


© 2001 Jerry Marlow
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250%
200%
141%
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50%
0%
-50%
-100%
-150%
-200%
to
-600%
to
-Inf%
Probability of Profit 0.34
Probability in the Money 0.41
Period Option Exp Return 59.7%
Equiv Simple Percent 81.6%
Annualized Period
Expected CC Return 15.00 7.5205%
Median CC Return 4.875% 2.4442%
Standard Deviation 45.00 31.8634%
No Dividend 0.00 0.00
Current Asset Price 100.00
Days to Expire (365 D/Y) 183
Call Strike Price 110.00
Call Option Price 7.00
-100%
-110%
-120%
Draw Your Forecast
Call
Simulate Price Change
Fast
Faster
Fastest
Draw Option Forecast
Hide Simple Percent
38.564%
Calculate Expected Return
Fast
Fastest
Color Deciles
308.064%

We can use color deciles to see how forecasts for stock return, stock price, and option return correspond to one another

The outcomes of a given color in one probability distribution correspond to the outcomes of the same color in the other distributions.

38. Click on Simulate Price Change until you get a few winners.

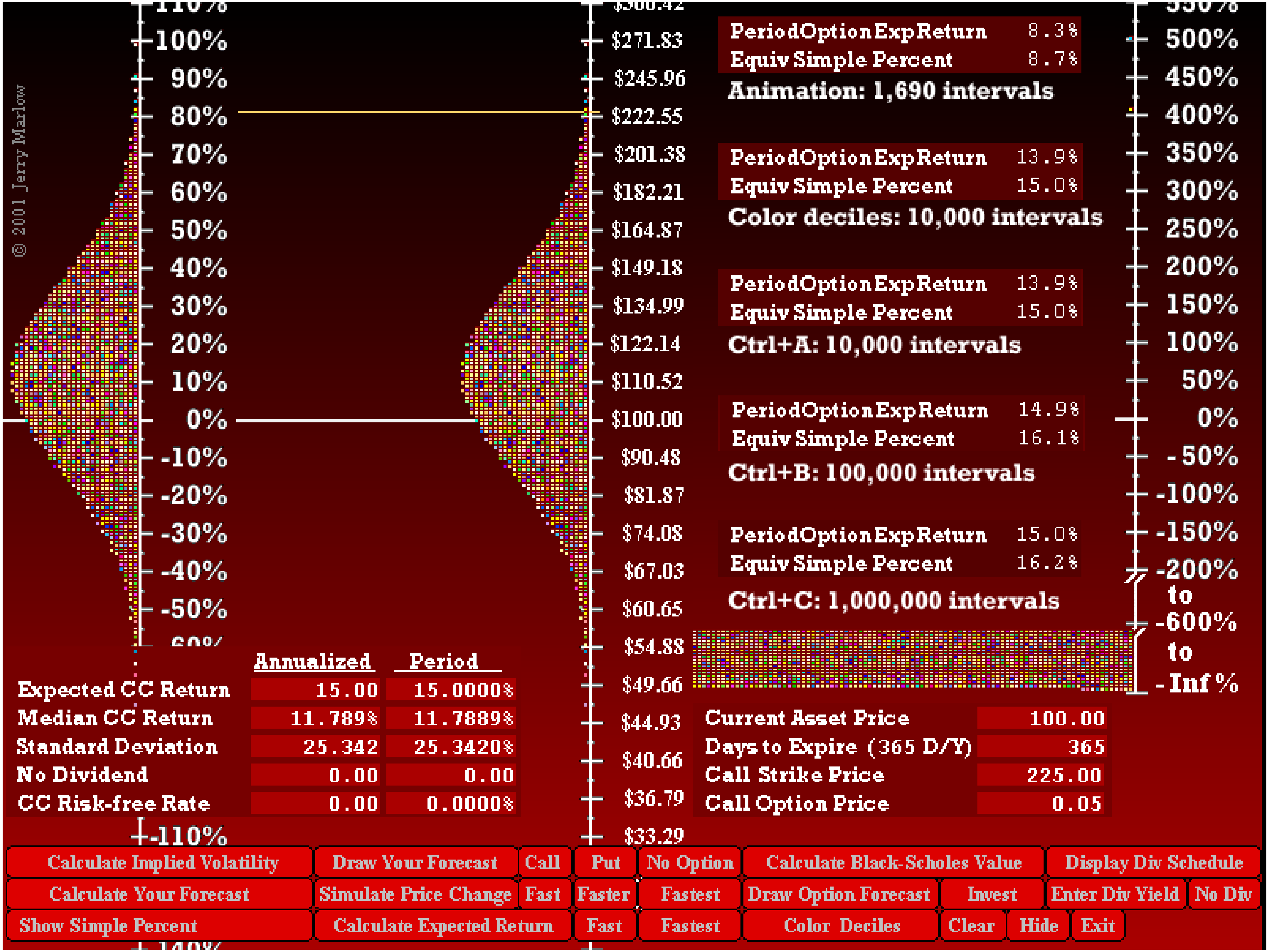
In general, the little squares that tabulate the outcomes appear in the same color in all three distributions.

The general statement is not precisely true for option returns that are below -200%.

The animation lumps together in one bucket all outcomes from -200% to -600%. A hit in this bucket means you've lost from 86.47% to 99.75% of your money. Because multiple deciles may show up in this bucket, the segregation of hits by color may not be maintained.

The animation lumps together in another bucket all outcomes between -600% and negative infinity. A hit in this bucket means you've lost between 99.75% and 100% of your money. Because multiple deciles usually show up in this bucket, the segregation of hits by color usually is not maintained.

Also, at the transition between any two deciles, the same bucket may contain two different colors. The tabulating square may end up in one color on one distribution and in a different color on another.


© 2001 Jerry Marlow
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$54.88
$49.66
$44.93
$40.66
$36.79
$33.29
PeriodOptionExpReturn 8.3%
Equiv Simple Percent 8.7%
Animation: 1,690 intervals
PeriodOptionExpReturn 13.9%
Equiv Simple Percent 15.0%
Color deciles: 10,000 intervals
PeriodOptionExpReturn 13.9%
Equiv Simple Percent 15.0%
Ctrl+A: 10,000 intervals
PeriodOptionExpReturn 14.9%
Equiv Simple Percent 16.1%
Ctrl+B: 100,000 intervals
PeriodOptionExpReturn 15.0%
Equiv Simple Percent 16.2%
Ctrl+C: 1,000,000 intervals
500%
450%
400%
350%
300%
250%
200%
150%
100%
50%
0%
-50%
-100%
-150%
-200% to -600% to -Inf %
Annualized
Period
Expected CC Return 15.00 15.0000%
Median CC Return 11.789% 11.7889%
Standard Deviation 25.342 25.3420%
No Dividend 0.00 0.00
CC Risk-free Rate 0.00 0.0000%
Current Asset Price 100.00
Days to Expire (365 D/Y) 365
Call Strike Price 225.00
Call Option Price 0.05
Calculate Implied Volatility
Draw Your Forecast
Call
Put
No Option
Calculate Black-Scholes Value
Display Div Schedule
Calculate Your Forecast
Simulate Price Change
Fast
Faster
Fastest
Draw Option Forecast
Invest
Enter Div Yield
No Div
Show Simple Percent
Calculate Expected Return
Fast
Fastest
Color Deciles
Clear
Hide
Exit

At extremes of probability distributions, Monte Carlo calculations are not very accurate. For greater accuracy, use Ctrl+A, B, or C.

When you click on Calculate Expected Return, Fast, or Fastest, the animation sweeps through the probability distribution divides it into 1,690 intervals, and calculates an outcome for each one.

For strike prices that are well within a stock's probability distribution, the sweep gives reasonably accurate results. If, however, you are working with a strike price at the extreme of a probability distribution, the 1,690 intervals may not be enough to give accuracy sufficient for your purposes.

39. Click on Clear.
40. For Days to Expire, enter 365.
41. For Call Strike Price, enter 225.00.
42. For Call Option Price, enter 0.05.
43. For Standard Deviation, enter 25.342.
44. Click on Calculate Expected Return Fastest.

The Monte Carlo sweep through the probability distribution calculates the period option expected return to be 8.3%. By other methods, the expected return of this option can be shown to be 15%.

Why the error?

You see that there are only a few outcomes in the probability distribution above the strike price of $225.00. With only a small region above the strike price, the 1,690 intervals that the animation uses are not enough to achieve accuracy.

To achieve better accuracy at extremes, professional derivative-pricing software that runs Monte Carlo simulations divides probability distributions into 10,000, 100,000, or perhaps even a million intervals.

Dividing a probability distribution into that many intervals is not feasible for animations that must run on computers of many different speeds. The animation does, however, include calculation routines that divide the probability distribution into 10,000, 100,000, and one million intervals. These routines do not animate.

45. Click on Color Deciles.

After drawing the color deciles, the routine divides the distribution into 10,000 intervals and calculates the expected return, in this case, 13.9%. Closer, but still not the right answer.

You also can divide the probability distribution into 10,00 intervals without drawing color deciles.

46. To have the Monte Carlo simulation divide the probability distribution into 10,000 intervals, hold down the Ctrl key on your keyboard and press the A key.

The routine calculates the period option expected return to be 13.9%.

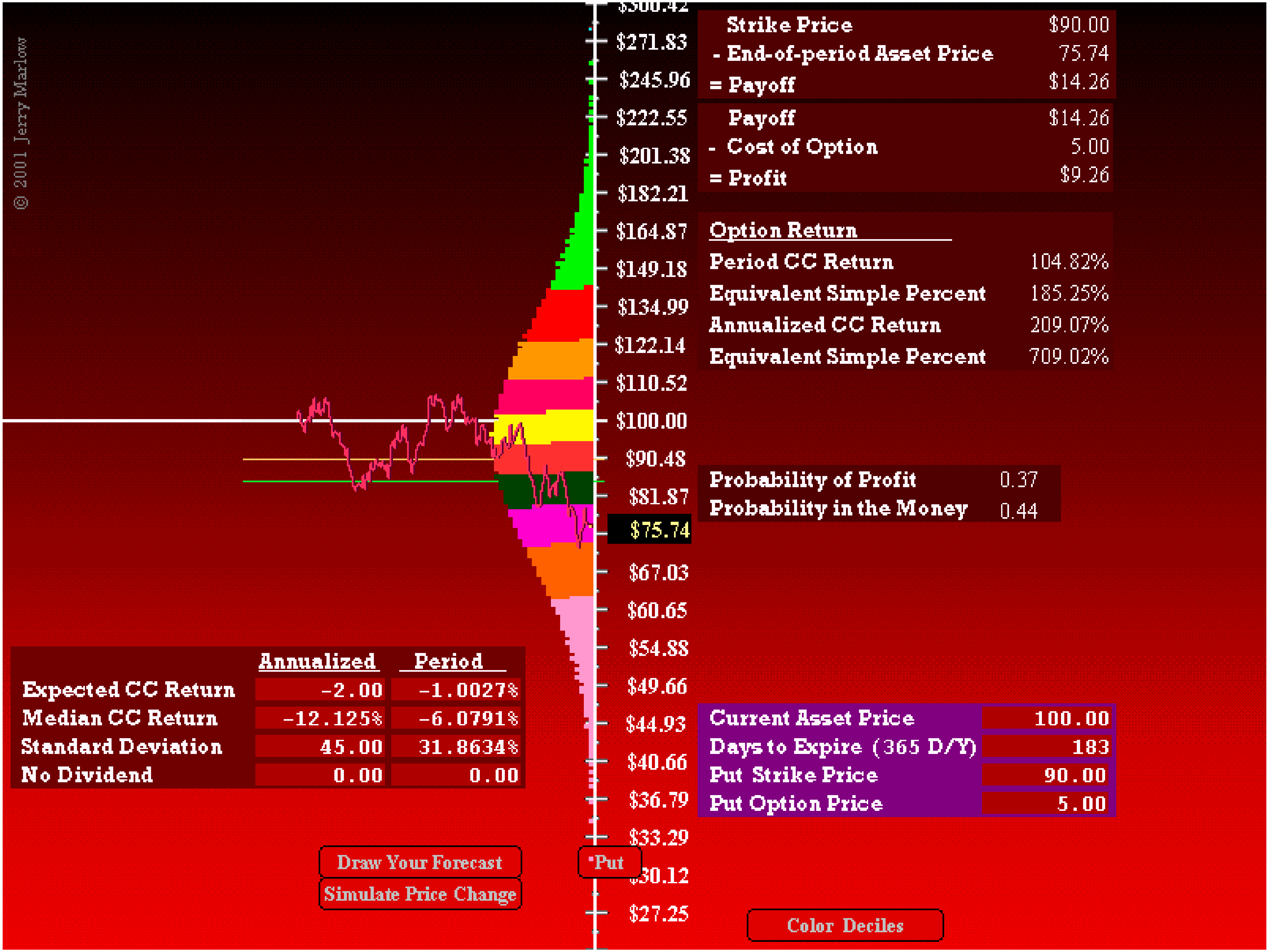
47. To divide the probability distribution into 100,000 intervals, hold down the Ctrl key and press the B key.

The routine calculates the period option expected return to be 14.9%. Very close.

48. To divide the probability distribution into 1,000,000 intervals, hold down the Ctrl key and press the C key.
49. Go have a sandwich.

By dividing the probability distribution into a million intervals, the Monte Carlo routine calculates the period option expected return to be 15.0%.

50. If a calculation or animation is taking too long, to interrupt it, click long and hard on any of the empty areas on the screen.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$300.42
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$67.03
$60.65
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$49.66
$44.93
$40.66
$36.79
$33.29
$30.12
$27.25
Strike Price $90.00
- End-of-period Asset Price 75.74
= Payoff $14.26
Payoff $14.26
- Cost of Option 5.00
= Profit $9.26
Option Return
Period CC Return 104.82%
Equivalent Simple Percent 185.25%
Annualized CC Return 209.07%
Equivalent Simple Percent 709.02%
Probability of Profit 0.37
Probability in the Money 0.44
Annualized Period
Expected CC Return -2.00 -1.0027%
Median CC Return -12.125% -6.0791%
Standard Deviation 45.00 31.8634%
No Dividend 0.00 0.00
Current Asset Price 100.00
Days to Expire (365 D/Y) 183
Put Strike Price 90.00
Put Option Price 5.00
Draw Your Forecast
Simulate Price Change
Put
Color Deciles

A put gives you the right to sell a stock at a pre-set price

When you buy a put option, you buy the right to sell a stock at a pre-set price called the strike price.

If the price of the stock is below the strike price or goes below the strike price and you exercise the option, you receive the difference between the strike price and the stock price at the time of exercise.

Given a stock forecast and a put, we can create the same types of animations that we created for call options.

We can simulate potential price paths for the stock and show the option's corresponding end-of-period payoffs.

1. Click three times on Clear.
2. Click twice on Hide.
3. Click on Simulate Price Change.
4. For Expected CC Return, enter –2.0.
5. For Standard Deviation, enter 45.
6. For Current Asset Price, enter 100.00.
7. Click on Put.
8. For Days to Expire, enter 183.
9. For Put Strike Price, enter 90.00.
10. For Put Option Price, enter 5.00.

The yellow line represents the option's strike price. For a put option to be in the money, the stock price must be below the yellow line.

The distance from the yellow line to the green line represents the option price or premium. For a put option to produce a profit at the time of exercise, the stock price must be below the green line.

11. Click on Color Deciles.

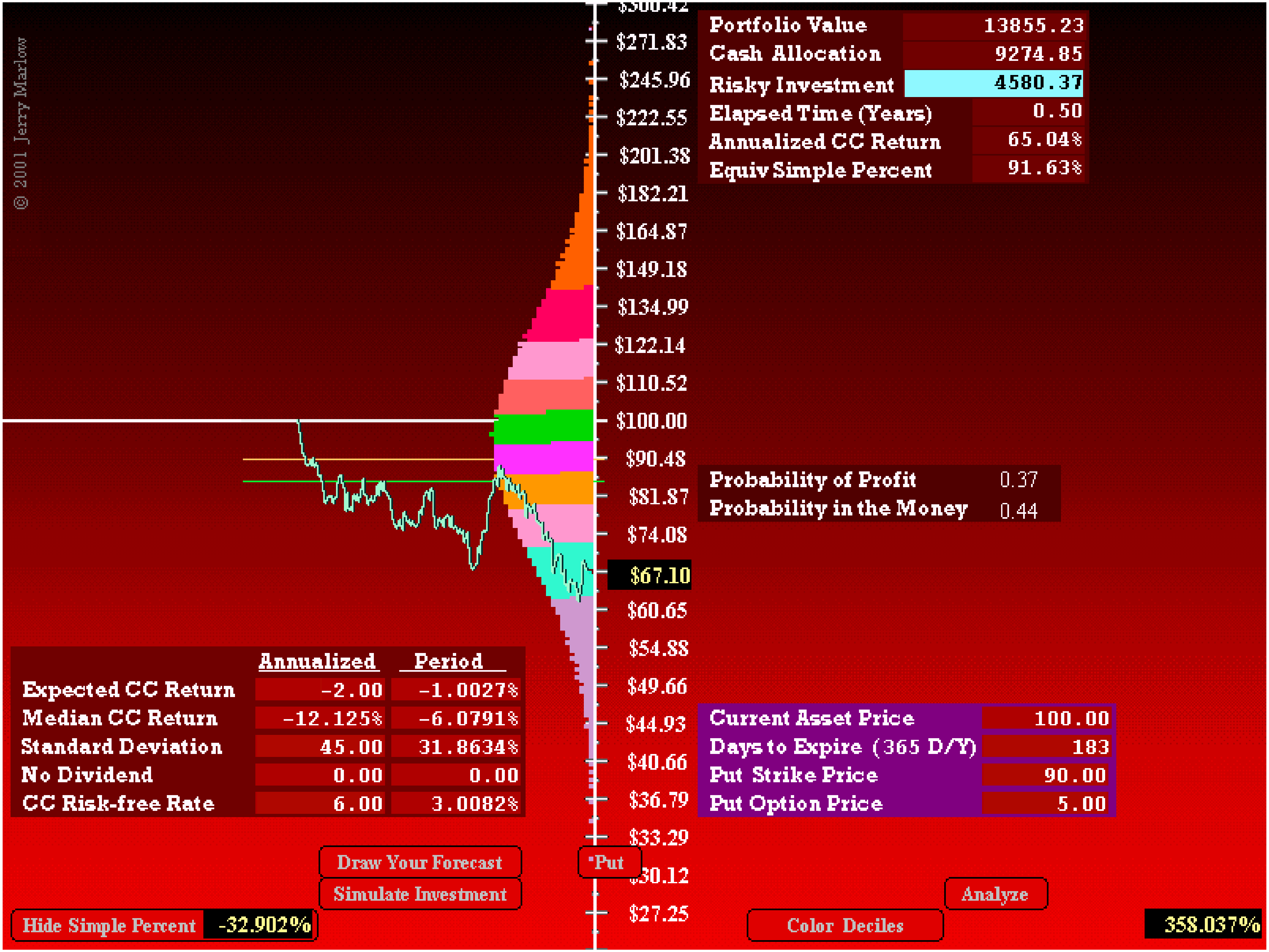
The area of the probability distribution below the yellow line represents the probability that the option will finish in the money.

The area of the probability distribution below the green line represents the probability that the option will produce a profit.

12. Click on Simulate Price Change. Keep clicking until you get a few winners.

When the end-of-period price is above the strike price, you see that the payoff is zero, your "profit" is negative, and you lose all the money you invested in the option. You have a period continuously compounded rate of return of negative infinity and a holding-period return of -100%.

When the end-of-period price is below the strike price, you see that the payoff is equal to the strike price minus the end-of-period price.



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Portfolio Value 13855.23
Cash Allocation 9274.85
Risky Investment 4580.37
Elapsed Time (Years) 0.50
Annualized CC Return 65.04%
Equiv Simple Percent 91.63%
$271.83
$245.96
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$74.08
$67.10
$60.65
$54.88
$49.66
$44.93
$40.66
$36.79
$33.29
$30.12
$27.25
Probability of Profit 0.37
Probability in the Money 0.44
Annualized Period
Expected CC Return -2.00 -1.0027%
Median CC Return -12.125% -6.0791%
Standard Deviation 45.00 31.8634%
No Dividend 0.00 0.00
CC Risk-free Rate 6.00 3.0082%
Current Asset Price 100.00
Days to Expire (365 D/Y) 183
Put Strike Price 90.00
Put Option Price 5.00
Draw Your Forecast
Simulate Investment
Put
Analyze
Color Deciles
Hide Simple Percent -32.902%
358.037%

Once you have your price forecast for a stock, you can simulate potential outcomes of investing in a put you hold to maturity

Let's say you have $10,000.00 in your portfolio. You want to invest $1,000.00 in the option.

13. Click on Show Simple Percent.
14. Click on Invest.
15. For CC Risk-free Rate, enter 6.0.
16. For Portfolio Value, enter 10000.00. (No $ or commas.)
17. For Risky Investment, enter 1000.00.
18. Click on Simulate Investment.

The animation displays the new value of the portfolio, the new value of the cash allocation, the proceeds of the investment in the option, the elapsed investment period in years, the annualized continuously compounded rate of return for the portfolio, and the equivalent simple percent.

To get a feel for how investing in put options can leverage your investment, make a few more investments of $1,000.00 in the option.

19. For Risky Investment, enter 1000.00.
20. Click on Simulate Investment.

If you wish to see only the effect on the portfolio of investing in the option, set the risk-free rate to zero.

21. For CC Risk-free Rate, enter 0.0.
22. For Portfolio Value, enter 10000.00. (No $ or commas.)
23. For Risky Investment, enter 1000.00.
24. Click on Simulate Investment.

Instead of displaying the option payoffs as numerical results, the animation can display option performance graphically as a return on your investment in the option.

25. To make the Invest data fields go away, click on Analyze.
26. To bring the option axes on screen, click once on Draw Option Forecast.
27. Click on Simulate Price Change.

To tabulate each option return, the animation adds a little square to the option axes.

You can simulate a price change and option return without drawing the price path.

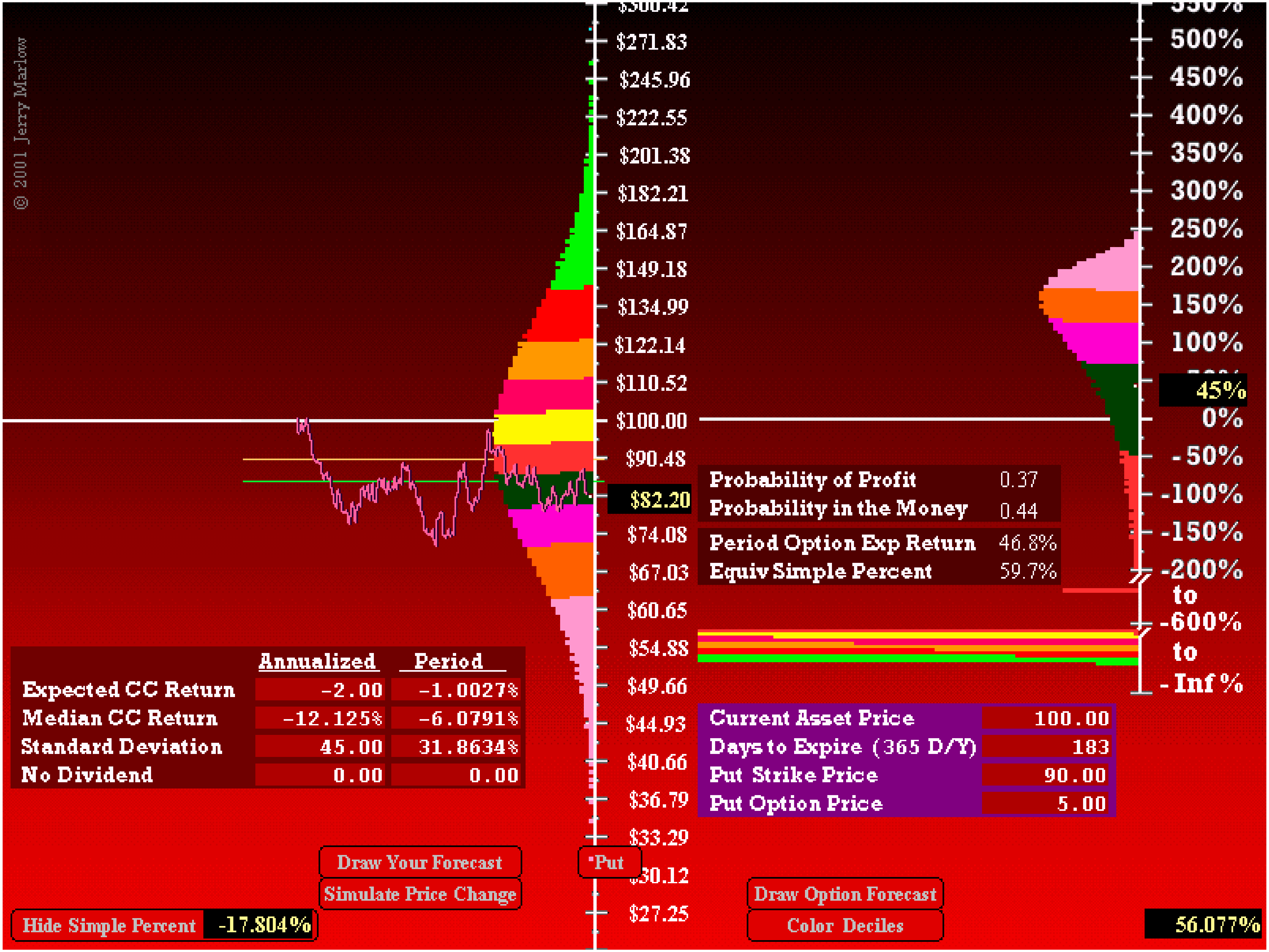
28. Click on Fast immediately to the right of Simulate Price Change.

You can speed up the simulation.

29. Click on Faster to the right of Simulate Price Change.

You can speed up the simulation further.

30. Click on Fastest to the right of Simulate Price Change.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45%
0%
-50%
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-150%
-200%
to
-600%
to
- Inf %
Probability of Profit 0.37
Probability in the Money 0.44
Period Option Exp Return 46.8%
Equiv Simple Percent 59.7%
Annualized Period
Expected CC Return -2.00 -1.0027%
Median CC Return -12.125% -6.0791%
Standard Deviation 45.00 31.8634%
No Dividend 0.00 0.00
Current Asset Price 100.00
Days to Expire (365 D/Y) 183
Put Strike Price 90.00
Put Option Price 5.00
Draw Your Forecast
Simulate Price Change
Put
Draw Option Forecast
Color Deciles
Hide Simple Percent -17.804%
56.077%

Given your price forecast for a stock, we can calculate a put's probability of profit and period expected return

The histogram of squares on the left of the screen approximates the probability distribution of the stock's price forecast. Given this stock forecast and option structure, the histogram formed on the option axes approximates the probability distribution or forecast of the option return.

For the put, we can calculate:

- The option's probability of profit
- The probability that the end-of-period stock price will be in the money
- The period expected return of the option
- The equivalent simple percent of the period option expected return

31. Click once on Clear.
32. Click a few times on Calculate Expected Return.

For period average return, the animation is simply calculating a running average of the option returns thus far swept through.

33. Click on Fast immediately to the right of Calculate Expected Return.

Automatically, the animation continues to sweep through the stock forecast, display the corresponding option outcomes, and tabulate the proportion of outcomes profitable and in the money.

34. Click on Fastest to the right of Calculate Expected Return.

The animation more quickly sweeps through the probability distributions.

When you click on Color Deciles, the animation divides each of the probability distributions into color-coded deciles.

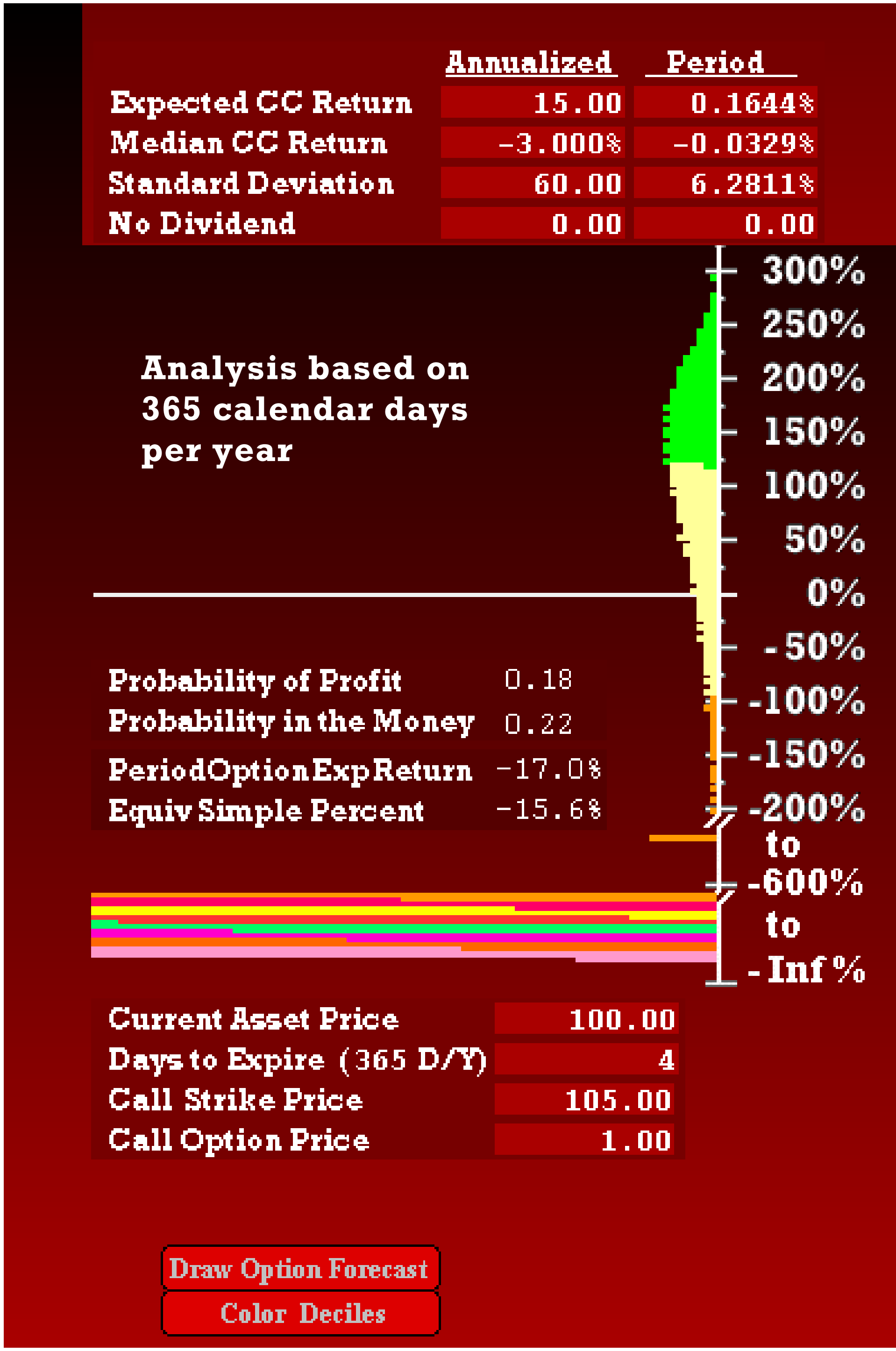
35. Click on Color Deciles.

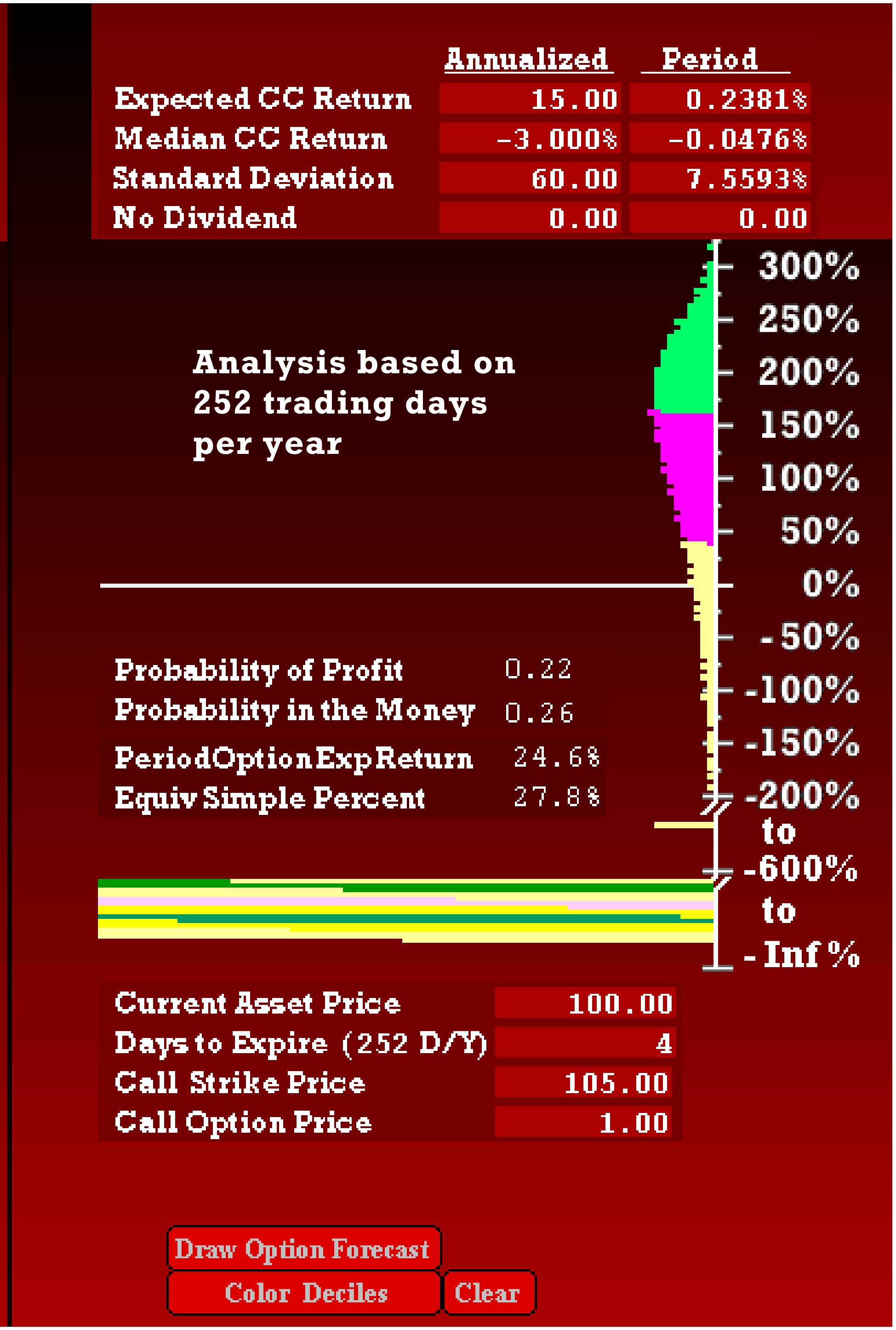
The outcomes of a given color in one probability distribution correspond to the outcomes of the same color in the other distributions.

After it draws the color deciles, the routine divides the probability distribution into 10,000 intervals and calculates the expected return and probability of profit.

36. Click on Simulate Price Change.

In general, the little squares that tabulate the outcomes appear in the same color in all three distributions.


Annualized Period
Expected CC Return 15.00 0.1644%
Median CC Return -3.000% -0.0329%
Standard Deviation 60.00 6.2811%
No Dividend 0.00 0.00
Analysis based on
365 calendar days
per year
300%
250%
200%
150%
100%
50%
0%
-50%
-100%
-150%
-200%
to
-600%
to
-Inf %
Probability of Profit 0.18
Probability in the Money 0.22
PeriodOptionExpReturn -17.0%
Equiv Simple Percent -15.6%
Current Asset Price 100.00
Days to Expire (365 D/Y) 4
Call Strike Price 105.00
Call Option Price 1.00
Draw Option Forecast
Color Deciles


Annualized Period
Expected CC Return 15.00 0.2381%
Median CC Return -3.000% -0.0476%
Standard Deviation 60.00 7.5593%
No Dividend 0.00 0.00
Analysis based on
252 trading days
per year
300%
250%
200%
150%
100%
50%
0%
-50%
-100%
-150%
-200%
to
-600%
to
-Inf %
Probability of Profit 0.22
Probability in the Money 0.26
PeriodOptionExpReturn 24.6%
Equiv Simple Percent 27.8%
Current Asset Price 100.00
Days to Expire (252 D/Y) 4
Call Strike Price 105.00
Call Option Price 1.00
Draw Option Forecast
Color Deciles
Clear

If you count days to expiration in trading days, set days per year to 252. Pay special heed if very few days are left until expiration.

One of the debates that people who trade options engage in is whether non-trading days account for some of a stock's volatility. People who believe non-trading days or events that occur on non-trading days account for some of a stock's volatility think of a year as having 365 days (366 in a leap year). They count days to expiration in calendar days.

People who believe only trading days matter think of a year as having 252 days. They count days to expiration in trading days.

The animation accommodates either belief. Whenever you have a call or put on screen, you can enter any number you want for days per year. Just click in or tab to the (____D/Y) field next to Days to Expire.

The animation uses the number in the days-per-year field and the number in the days-to-expire field to calculate the length of the current period as a fraction of a year. It then uses this fraction-of-a-year number to calculate period and annualized numbers from one another.

In particular, the animation uses the period length to calculate period numbers for expected return, median return, standard deviation, risk-free rate, and dividend yield. It then uses these number to do analysis and run simulations.

For long times to expiration, whether you use a 365- or 252-day year doesn't have much practical consequence. For example, 183 days of a 365-day year gets pretty much the same results as 126 days of a 252-day year. The important thing is to keep the way you count days to expiration consistent with the number of days you entered for days per year.

When you get down to short times to expiration, however, especially times in the vicinity of a week or less, the number in the days-per-year field and how you count days to expiration can make a big difference.

Let's say it's Monday. You're looking at an option that expires on Friday.

1. Click three times on Clear.
2. Click twice on Hide.
3. Click on Call.
4. Click on Draw Option Forecast.
5. For Current Asset Price, enter 100.00.
6. For (____D/Y), enter 365.
7. For Days to Expire, enter 4.
8. For Call Strike Price, enter 105.00.
9. For Call Option Price, enter 1.00.
10. For Expected CC Return, enter 15.
11. For Standard Deviation, enter 60.

Make note of the period figures for expected return, median return, and standard deviation.

12. Click on Color Deciles.

The animation calculates probability of profit as 0.18 and period option expected return as -17.0%.

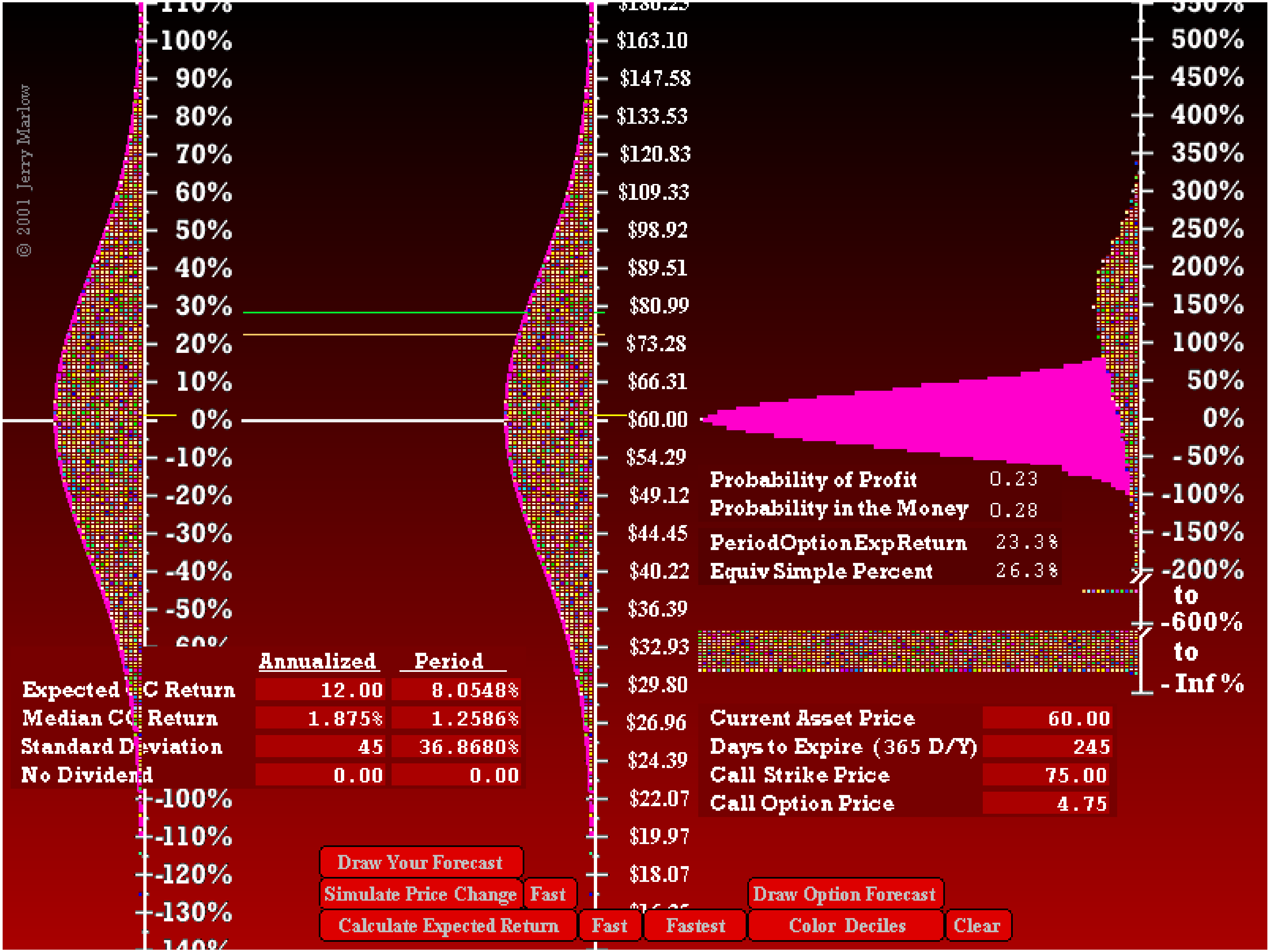
13. For (____D/Y), enter 252.

You see that the period figures for expected return, median return, and standard deviation all increase.

14. Click on Color Deciles.

The animation calculates probability of profit as 0.22 and period option expected return as 24.6%. Big difference!

Figure out whether you believe in calendar days or trading days. For number of days per year, enter 365 or 252. Count days to expiration accordingly.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$73.28
$66.31
$60.00
$54.29
$49.12
$44.45
$40.22
$36.39
$32.93
$29.80
$26.96
$24.39
$22.07
$19.97
$18.07
500%
450%
400%
350%
300%
250%
200%
150%
100%
50%
0%
-50%
-100%
-150%
-200%
to
-600%
to
-Inf %
Probability of Profit 0.23
Probability in the Money 0.28
PeriodOptionExpReturn 23.3%
Equiv Simple Percent 26.3%
Annualized
Period
Expected CC Return 12.00 8.0548%
Median CC Return 1.875% 1.2586%
Standard Deviation 45 36.8680%
No Dividend 0.00 0.00
-100%
-110%
-120%
-130%
Current Asset Price 60.00
Days to Expire (365 D/Y) 245
Call Strike Price 75.00
Call Option Price 4.75
Draw Your Forecast
Simulate Price Change
Fast
Draw Option Forecast
Calculate Expected Return
Fast
Fastest
Color Deciles
Clear

Does the expression probability density function *make your brain hurt?*

Mathematicians often refer to probability distributions as probability density functions.

What the heck does that mean?

Then they start talking about the area under the curve.

What are they talking about?

Let's see.

1. Click three times on Clear.
2. Click twice on Hide.
3. Click on Simulate Price Change.
4. Click on Draw Your Forecast.
5. Click on Call.
6. Click on Draw Option Forecast.
7. For Current Asset Price, enter 60.00.
8. For Days to Expire, enter 245.
9. For Call Strike Price, enter 75.00
10. For Call Option Price, enter 4.75.
11. If, in the previous exercise you changed days per year, for (___ D/Y), enter 365.
12. For Expected CC Return, enter 12.00.
13. For Standard Deviation, enter 45.00.
14. Click on Draw Your Forecast.
15. Click on Draw Option Forecast.

The idea of a probability density function is that the total area enclosed by a bell-shaped curve adds up to one. (Some like to say it adds up to unity.) Similarly, the total area defined by an option forecast adds up to one.

To make the idea of area under the curve more obvious, we've been sweeping through probability distributions with little squares.

16. Click on Calculate Expected Return Fastest.

We've used these little squares to build histograms and simulate outcomes.

The way we've been doing this may've given you the impression that we can calculate the probability of a particular outcome, say the probability of a stock return of 40% or an end-of-period price of $89.51.

In a formal sense, we cannot assign a probability to a particular outcome. We can only compute probabilities for intervals.

Hence, we cannot assign a probability to an outcome of 40%. We can only compute the probability of an outcome between, say 39.5% and 40.5%.

Likewise, we cannot assign a probability to an outcome of $89.51. We can only compute the probability of an outcome between, say, $89.505 and $89.515.

So when you see a bunch of little squares stacked up next to 40%, keep in mind that they're really covering an interval between 39.5% and 40.5%. In the end-of-period price histograms above, the row of little squares adjacent to $89.51 covers the interval between $89.063 and $89.958.

The probability of outcomes in any one interval corresponds to the percentage of little squares that fall in that interval.

17. Click on Color Deciles.

If ten percent of the little squares fall in the area between returns of 10% and 20%, then the probability of a return between 10% and 20% is 0.1. If twenty percent of the area under the curve falls above an end-of-period price of $81.87, then the probability of an end-of-period price above $81.87 is 0.2.

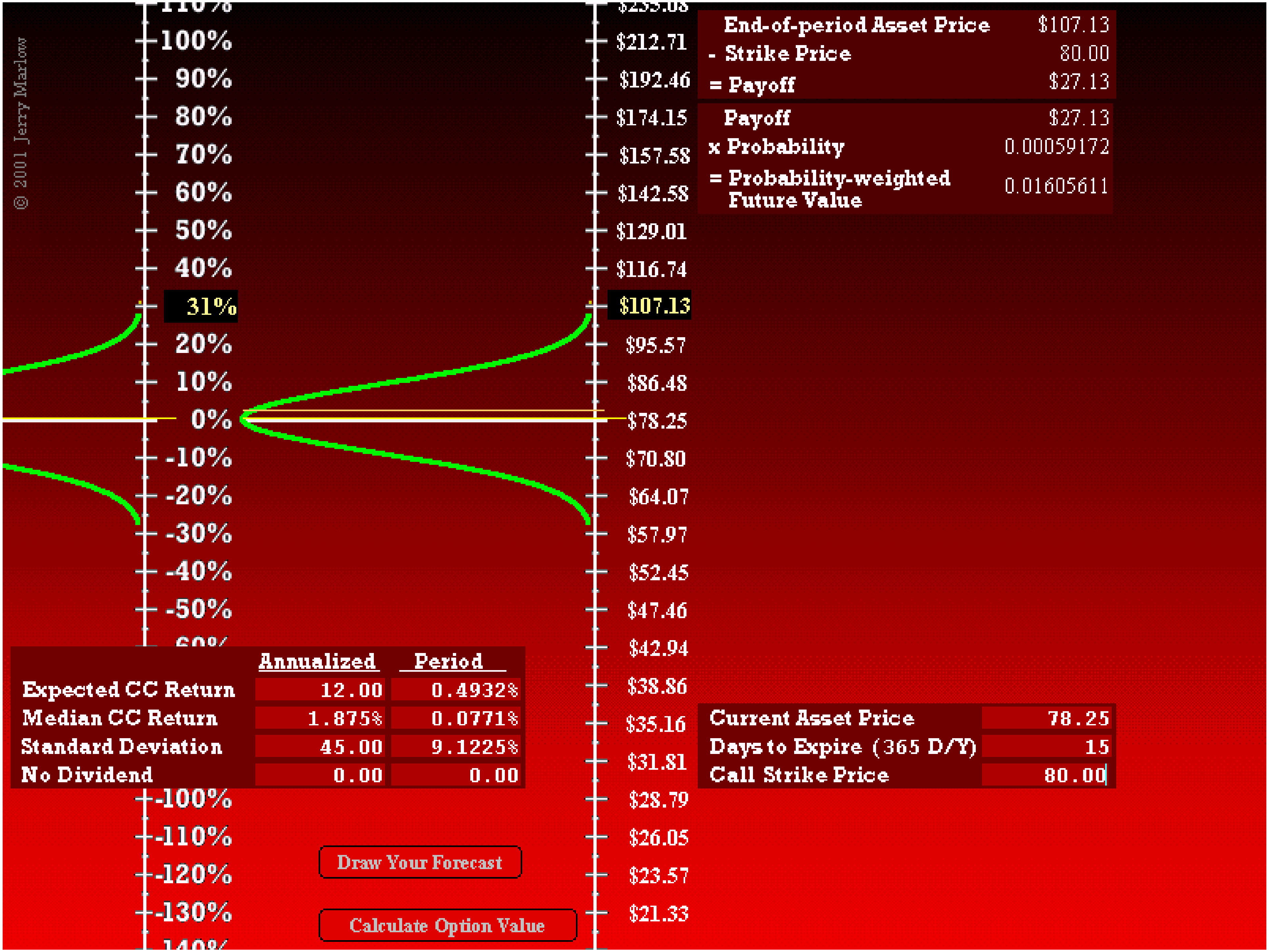
Now you understand probability density functions. Now you can make other peoples' brains hurt.

Option Pricing

Making options valuation intuitively accessible

For the non-mathematician, the mathematics of options valuation can be daunting. To make options valuation intuitively accessible, we look at options valuation in four steps:

1. Using your forecast for a stock, calculate the probability-weighted net present value of an option on the stock when the option is held to maturity.
2. Examine the Black-Scholes methodology for calculating the value of a European option.
3. Look at the assumption that underlies the Black-Scholes methodology.
4. See how to make an option's probability-weighted net present value equal its Black-Scholes value.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$70.80
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$52.45
$47.46
$42.94
$38.86
$35.16
$31.81
$28.79
$26.05
$23.57
$21.33
End-of-period Asset Price $107.13
- Strike Price 80.00
= Payoff $27.13
Payoff $27.13
x Probability 0.00059172
= Probability-weighted Future Value 0.01605611
Annualized Period
Expected CC Return 12.00 0.4932%
Median CC Return 1.875% 0.0771%
Standard Deviation 45.00 9.1225%
No Dividend 0.00 0.00
Current Asset Price 78.25
Days to Expire (365 D/Y) 15
Call Strike Price 80.00
Draw Your Forecast
Calculate Option Value

What's an option worth to you?
Its probability-weighted net present value

Let's begin with a stock forecast and a call option. From these, we'll calculate the probability-weighted net present value of the option when it is held to maturity.

1. Click three times on Clear.
2. Click twice on Hide.
3. Click on Calculate Option Value.
4. For Expected CC Return, enter 12.00.
5. For Standard Deviation, enter 45.00.
6. For Current Asset Price, enter 78.25.
7. For Days to Expire, enter 15.
8. For Call Strike Price, enter 80.00.
9. Click on Draw Your Forecast.

What we're going to do is sweep through the probability distribution of the stock forecast and calculate the probability-weighted net present value of all the option payoffs.

10. Click on Calculate Option Value.

The animation begins with the extreme high-price outcome for the stock forecast. From this outcome, it calculates the payoff.

If the stock price were to go to $107.13 and you exercised the option, your payoff would be $27.13.

11. Click again on Calculate Option Value.

Very often, financial calculations make use of the concept of probability-weighted value. If your brother-in-law owes you $500 and there's only one chance in ten that he is going to pay you, the probability-weighted value of the obligation is ($500)(1/10) or $50. If you believed that this was in fact the case, you might be willing to sell the obligation to a third party for $50.

Similarly, you might buy a raffle ticket. Let's say your local community organization is raffling off a new car. The car's fair market value is $39,375. The community organization will sell 100,000 tickets. The fair value of one ticket on the day of the raffle is $39,375 times 1/100,000 or $0.39.

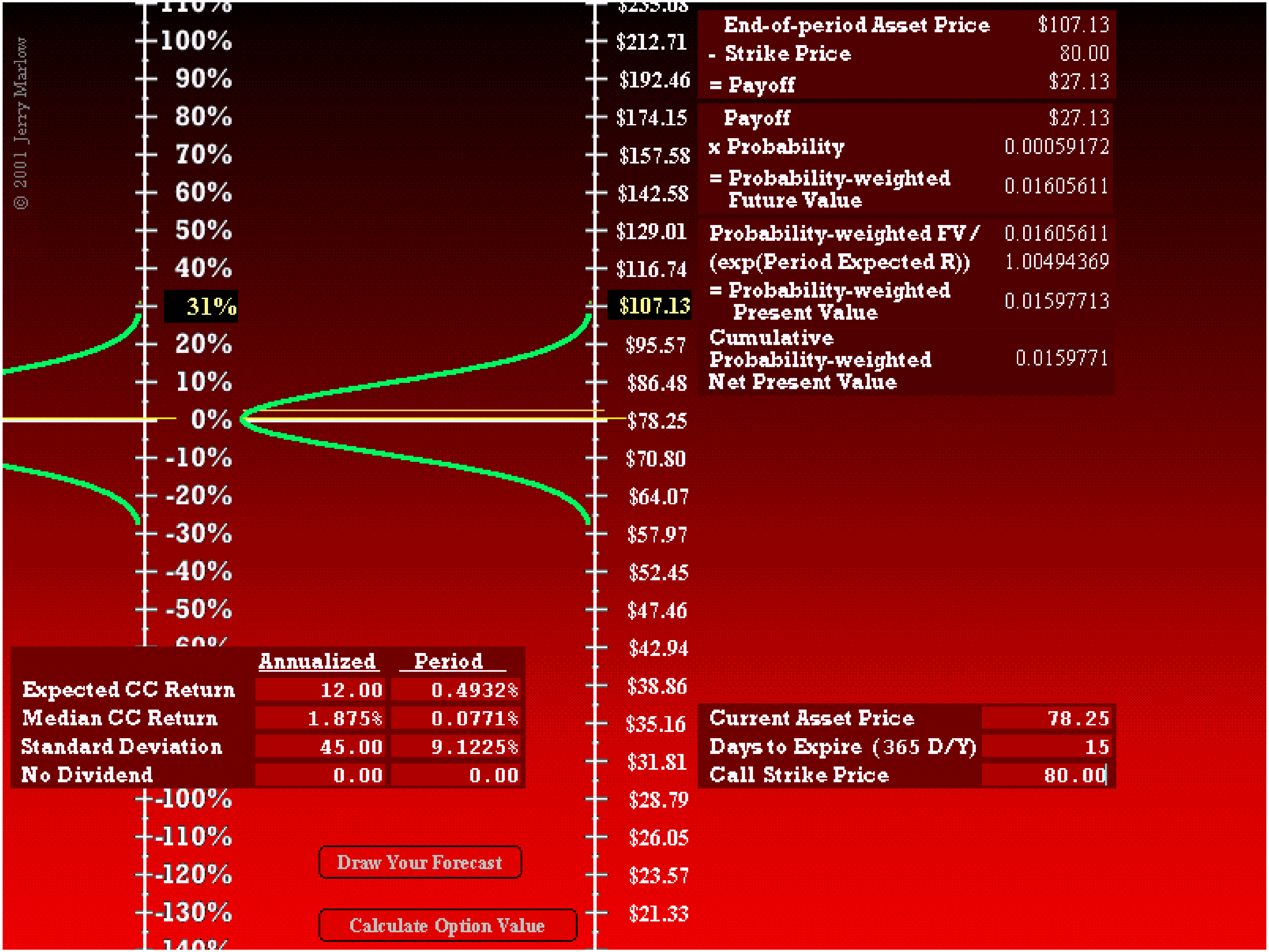
We apply the same concept to find the probability-weighted future value of the stock-price outcome of $107.13.

Given the scale at which the animation draws probability distributions, it takes 1,690 little squares to fill in a probability distribution. Hence, the probability of any one representative outcome is 1/1,690 or .00059172.

When we multiply $107.13 times .00059172, we get $0.01605611—slightly more than 1.6 cents.

This is the fair value 15 days from now of this one outcome represented by the little square at $107.13. Hence, what we have calculated is the probability-weighted future value of this one outcome.

Next we want to calculate the probability-weighted present value of this one outcome.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$35.16
$31.81
$28.79
$26.05
$23.57
$21.33
End-of-period Asset Price $107.13
- Strike Price 80.00
= Payoff $27.13
Payoff $27.13
x Probability 0.00059172
= Probability-weighted Future Value 0.01605611
Probability-weighted FV / 0.01605611
(exp(Period Expected R)) 1.00494369
= Probability-weighted Present Value 0.01597713
Cumulative Probability-weighted Net Present Value 0.0159771
Annualized Period
Expected CC Return 12.00 0.4932%
Median CC Return 1.875% 0.0771%
Standard Deviation 45.00 9.1225%
No Dividend 0.00 0.00
Current Asset Price 78.25
Days to Expire (365 D/Y) 15
Call Strike Price 80.00
Draw Your Forecast
Calculate Option Value

Finding the probability-weighted net present value of an option's probability distribution is like doing discounted cash-flow analysis in corporate finance

12. Click again on Calculate Option Value.

Earlier, we touched upon the concepts of present value and future value. You can multiply a present value by 1 plus the simple interest rate to get a future value.

If you buy a certificate of deposit (CD) today for $100 and it pays 10% simple interest, its future value one year from today is $110.

($100)(1.10) = $110.00.

Or you can go the other way. To get a present value, you discount the future value. You divide it by 1 plus the simple interest rate.

If a CD will be worth $110 a year from today and it pays 10% simple interest, its present value is $100.

$110.00/1.10 = $100.00.

The relationship between continuously compounded rates of return and simple interest rates is:

1 + simple interest rate = exp(continuously compounded rate of return)

To discount the probability-weighted future value of an option outcome, we use the period expected continuously compounded rate of return. In our example,

1 + simple interest rate = exp(.004932)
= 1.00494369

Therefore, in our example, the probability-weighted net present value (PWNPV) of the one outcome of $107.13 is

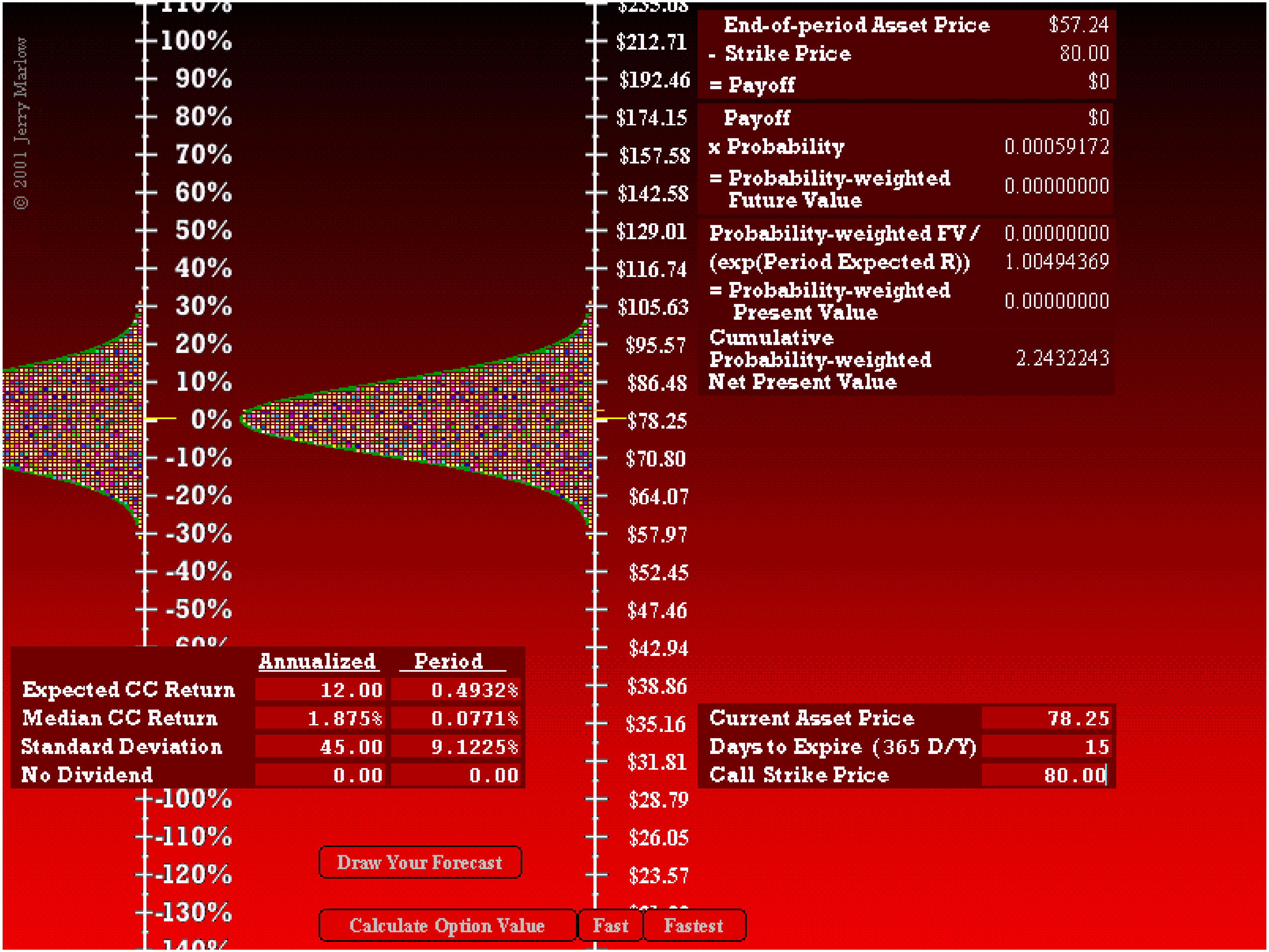
PWNPV = probability weighted future value / exp(continuously compounded rate of return)
= $0.01605611/1.00494369
= $0.01597713

We have calculated the probability-weighted net present value of one outcome. If you were going to buy a raffle ticket on this one outcome, the raffle ticket's fair value would be $0.01597713 or slightly less than 1.6 cents.

13. Click again on Calculate Option Value.

Here we save the $0.01597713 as the first installment of the cumulative probability-weighted net present value for the entire probability distribution.

14. Click a few more times on Calculate Option Value.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$42.94
$38.86
$35.16
$31.81
$28.79
$26.05
$23.57
End-of-period Asset Price $57.24
- Strike Price 80.00
= Payoff $0
Payoff $0
x Probability 0.00059172
= Probability-weighted Future Value 0.00000000
Probability-weighted FV / 0.00000000
(exp(Period Expected R)) 1.00494369
= Probability-weighted Present Value 0.00000000
Cumulative Probability-weighted Net Present Value 2.2432243
Annualized Period
Expected CC Return 12.00 0.4932%
Median CC Return 1.875% 0.0771%
Standard Deviation 45.00 9.1225%
No Dividend 0.00 0.00
Current Asset Price 78.25
Days to Expire (365 D/Y) 15
Call Strike Price 80.00
Draw Your Forecast
Calculate Option Value
Fast
Fastest

If you agree with the forecast and you do not require extra compensation for taking on the exposure to uncertainty, you might be willing to pay for an option its probability-weighted net present value

The animation goes through the same series of calculations for each potential outcome and adds the probability-weighted net present value to the cumulative probability-weighted net present value.

15. Click on Fast immediately to the right of Calculate Option Value.

The animation automatically continues to sweep through the stock forecasts and calculate the probability-weighted net present value for the potential outcomes.

16. Click on Fastest to the right of Calculate Option Value.

The animation more quickly sweeps through the forecasts. When it finishes, it displays the cumulative probability-weighted net present value of the outcomes in the forecast.

If you agreed with the stock forecast and you did not require extra compensation for taking on the risk associated with an investment in the option, then you might reasonably accept that the probability-weighted net present value, $2.24, is fair value for this option.

The Black-Scholes value of an option is the expected cost of hedging the sale of the option

In the previous section, we took a forecast for a stock and combined it with a call option. We calculated the probability-weighted net present value of the option held to maturity. Is this the price you would have to pay in the marketplace to buy this option?

No. Not if the risk-free rate is less than your expected return of 12%.

Why not?

If you were to pay $2.24 for the option in the previous example, an options trader could sell you the option and simultaneously hedge his exposure at a cost of less than $2.24. He would be able to perform arbitrage. He would be able to earn a risk-free profit.

Efficient financial markets quickly eliminate opportunities for arbitrage. Therefore, the fair price of an option is the cost to a trader of hedging the exposure he creates when he sells an option.

The Black-Scholes formula for the value of an option gives the cost of hedging the sale of the option. The formulas rely on the Black-Scholes assumptions about the behavior of the financial markets.

In theory, after a trader sets up a hedge, delta hedging keeps his exposures in balance at no additional costs. Hence, the Black-Scholes value is the cost of setting up the hedge.

When a person hedges exposures, he balances his assets and liabilities in such a way that an increase in his assets offsets any increase in his liabilities. A decrease in his liabilities offsets any decrease in his assets. If he keeps himself perfectly hedged, his overall exposure remains neutral. He shouldn't care whether his assets and liabilities grow or shrink so long as they stay in synch.

The Black-Scholes formulas rely on a form of hedging known as delta hedging. An option's delta is the ratio of option-price change to stock-price change. In delta hedging, a trader sets up the hedge so that his assets and liabilities are in balance.

To hedge the sale of a call, the assets and liabilities a trader balances are these:

Assets

- Delta shares of stock

Liabilities

- A bond that grows or shrinks in value with the probability that the option is going to finish in the money
- The value of the options that he has sold

As the number of days to expiration decreases and the stock price changes, the trader keeps his exposures in balance. Sometimes he borrows more money and buys more stock. Sometimes he sells stock and pays down some of the bond.

When the option expires, it is either in the money or out of it. If the trader has sold call options and the options expire in the money, at the time of expiration, he will own enough shares to satisfy the call. Receiving the strike price for the shares will enable him to pay off the bond.

If the call option expires out of the money, at the time of expiration, the trader will hold no shares. The value of the bond will have gone to zero, and he will owe no money on it.

Let's look at the delta hedging assets and liabilities in greater detail.

Number of options	10,000
Strike price	$80.00
Standard deviation	45.0%
CC Risk-free rate	5.0%
Dividends	$0.00

Daily Delta Hedging In Concept—Matching Assets to Liabilities

Assets							
Days to Exp	Stock Price	Option Delta	Shares Required for Delta Hedge—Delta X Number of Options	Shares Purchased or Sold to Meet Requirement	Shares Held after Purchase or Sale	Cost of Shares Purchased or Sold	**Total Assets—Market Value of Shares Held**
15	$78.25	0.43081	4,308	4,308	4,308	$337,107	**$337,107**
14	$82.52	0.66193	6,619	2,311	6,619	$190,722	**$546,225**
13	$81.74	0.62430	6,243	-376	6,243	-$30,759	**$510,303**
12	$79.51	0.49427	4,943	-1,300	4,943	-$103,387	**$392,994**
11	$78.67	0.43792	4,379	-564	4,379	-$44,331	**$344,512**
10	$80.28	0.54084	5,408	1,029	5,408	$82,624	**$434,186**
9	$82.25	0.67195	6,720	1,311	6,720	$107,838	**$552,679**
8	$80.00	0.51984	5,198	-1,521	5,198	-$121,688	**$415,872**
7	$77.36	0.31138	3,114	-2,085	3,114	-$161,265	**$240,884**
6	$77.63	0.31628	3,163	49	3,163	$3,804	**$245,528**
5	$78.14	0.34189	3,419	256	3,419	$20,012	**$267,153**
4	$77.89	0.29729	2,973	-446	2,973	-$34,739	**$231,559**
3	$79.29	0.42542	4,254	1,281	4,254	$101,594	**$337,316**
2	$80.08	0.52189	5,219	965	5,219	$77,253	**$417,930**
1	$82.14	0.87251	8,725	3,506	8,725	$287,999	**$716,680**
0	$83.06	1.00000	10,000	1,275	10,000	$105,893	**$830,600**

Assets:
■ *Delta shares of stock*

An option's delta or hedge ratio is the ratio of a change in the option price to a change in the price of the underlying stock. For example, if a $1 change in the price of the stock will cause a $0.43081 change in the value of a call option, then the option's delta is 0.43081.

An option's delta remains the same only over a small price range for the stock. As the price of the stock changes, delta changes. Also, as the number of days to expiration changes, an option's delta changes.

A call option's delta always lies somewhere between 1.0 and 0. If a call is deep in the money and little time is left before it expires, the option's delta will be close to 1.0. A $1 change in the price of the stock will cause close to a $1 change in the value of the call.

If a call is deep out of the money and little time is left before the option expires, its delta will approach 0. A $1 change in the price of the stock will cause almost zero change in the value of the option.

To set up a delta hedge, a trader calculates the option's delta and buys delta shares of stock. Let's say the stock price is $78.25. The option's delta is 0.43081. The trader has sold 10,000 options.

To set up the hedge, the trader buys 4,308.1 shares of stock. He pays (4,308.1)($78.25) = $337,107.20 for the stock. At this point in time, his assets are 4,308.1 shares of stock with a market value of $337,107.20.

(To make the numbers come out right, we say the trader buys 4,308.1 shares of stock. In reality, the trader would buy 4,308 shares. Also 0.43081 is a rounded number. To perform the multiplication, we use the unrounded value and get the product $337,107.20. If you multiply 4,308.1 times $78.25, you get $337,108.825.)

To stay perfectly hedged, as the number of days to expiration and the stock price keep changing, a trader keeps recalculating the option's delta. When delta goes up, he borrows more money and buys more stock. When delta goes down, he sells off stock and pays down some of the bond.

At expiration, delta will be either one or zero. If a call option finishes in the money, delta will be one. The trader will own one share of stock for every option he has sold. He will hold the number of shares he needs to satisfy exercise of the call.

If the option finishes out of the money, delta will be zero. The trader will own no stock. The holder of the option will not exercise it.

The table above shows daily price changes for a stock over fifteen days. For each day and price, the table shows the delta for a call option on the stock and the shares purchased or sold to meet the requirements of a delta hedge.

Liabilities:

- ***A bond that grows or shrinks in value with the probability that the option is going to finish in the money***
- ***The value of the option***

To buy delta shares of stock, a trader raises money in two ways:

- At the risk-free rate, he sells a bond short. That is, he borrows money.
- He receives money from the sale of the options.

These actions create his liabilities.

How much money should he borrow?

Black-Scholes Options-Pricing Theory says the amount he should borrow is determined primarily by two factors: the probability that the option will finish in the money and the strike price.

Earlier we saw that, given a stock forecast and an option structure, the animation can calculate the probability that the option will finish in the money. As the days to expiration and the stock price change, the probability that an option will finish in the money changes.

The probability always lies somewhere between zero and one. If a call is deep in the money and little time is left before expiration, the probability that the option will finish in the money is close to one.

If an option is deep out of the money and little time is left before expiration, the probability is close to zero.

To set up a delta hedge for a call option, a trader calculates the probability that the option will finish in the money. He multiplies the probability times the option strike price. This calculation gives the probability-adjusted value of the bond at the time of the option's expiration.

Daily Delta Hedging In Concept—Matching Assets to Liabilities

					Liabilities					
Days to Exp	Stock Price	Value of Option	Probability Option will Finish in the Money	Value of Bond at Expiry—Probability In-Money X Strike Price	Hedge Require-ment—Bond Value Today	One Day Interest on Previous Day's Total Bond	Money Borrowed or Repaid to Meet Hedge Requirement	Total Bond Obligation—Borrowing + Interest	Value of Options Sold	**Total Liabilities—Borrowings + Interest due + Market Value of Options Sold**
15	$78.25	$2.152	0.39530	$316,237	$315,588	$0.00	$315,588	$315,588	$21,520	**$337,107**
14	$82.52	$4.387	0.62915	$503,320	$502,356	$43.23	$186,725	$502,356	$43,868	**$546,224**
13	$81.74	$3.780	0.59168	$473,344	$472,502	$68.82	-$29,923	$472,502	$37,801	**$510,303**
12	$79.51	$2.418	0.46178	$369,425	$368,818	$64.73	-$103,748	$368,818	$24,179	**$392,997**
11	$78.67	$1.912	0.40735	$325,878	$325,387	$50.53	-$43,481	$325,387	$19,122	**$344,509**
10	$80.28	$2.579	0.51119	$408,953	$408,393	$44.58	$82,961	$408,393	$25,790	**$434,183**
9	$82.25	$3.649	0.64604	$516,829	$516,193	$55.95	$107,743	$516,193	$36,486	**$552,679**
8	$80.00	$2.169	0.49327	$394,619	$394,187	$70.72	-$122,077	$394,187	$21,688	**$415,874**

7	$77.36	$0.935	0.28971	$231,764	$231,542	$54.00	-$162,699	$231,542	$9,346	**$240,887**
6	$77.63	$0.889	0.29604	$236,829	$236,635	$31.72	$5,061	$236,635	$8,890	**$245,525**
5	$78.14	$0.912	0.32276	$258,211	$258,035	$32.42	$21,367	$258,035	$9,117	**$267,152**
4	$77.89	$0.673	0.28119	$224,949	$224,826	$35.35	-$33,244	$224,826	$6,733	**$231,559**
3	$79.29	$0.985	0.40950	$327,598	$327,463	$30.80	$102,607	$327,463	$9,854	**$337,317**
2	$80.08	$1.115	0.50861	$406,885	$406,774	$44.86	$79,265	$406,774	$11,152	**$417,926**
1	$82.14	$2.275	0.86753	$694,025	$693,930	$55.73	$287,101	$693,930	$22,752	**$716,683**
0	$83.06	$3.060	1.00000	$800,000	$800,000	$95.07	$105,975	$800,000	$30,600	**$830,600**

For example:

	Probability Finish In the Money	0.39530
X	Strike Price	\$80.00
=	Bond Value at Expiration	\$31.6237

The trader calculates the present value of this amount.

	Bond Value at Expiration	\$31.6237
÷	exp(Period Risk-free rate)	1.00206
=	Bond Value Today	\$31.5588

For each option he sells, this is the amount the trader borrows. Had he sold 10,000 options, he would borrow (10,000)(\$31.5588) = \$315,588. This would be the amount of one of his liabilities when he sets up the hedge.

The trader's other liability is the value of the option. For now, let's just note that the value of the option at the time of setting up the hedge is \$2.15197. Had he sold 10,000 options, the liability of the options would be \$21,519.70.

Hence, at set up, for the sale of 10,000 options, we have:

Assets	
Market value of delta shares of stock	\$337,107.20
Liabilities	
Present value of bond obligation	\$315,587.50
Value of options sold	21,519.70
Total liabilities	\$337,107.20

As the time to expiration and the stock price change, the probability that the option will finish in the money changes. To remain hedged, the trader recalculates the probability that the option will finish in the money and recalculates the present value of the hedge bond. The value of the options sold changes. The hedge stays in balance.

At expiration, the probability that the option will finish in the money is either one or zero. If a call option finishes in the money, the probability is one. The value of the bond will be equal to the strike price times the number of options sold.

When the option holder exercises the option, the trader receives the strike price times the number of options sold. He uses the proceeds to pay off the bond.

If a call option finishes out of the money, the final probability of finishing in the money is zero. The bond value is zero. The option holder does not exercise the option. The trader receives no money. The trader has no bond to pay off.

The table on page 134 shows the asset side of a delta hedge. The table above shows the liabilities side of the same hedge. For each day and price, the table shows the probability the option will finish in the money, the bond value, option value, and total liabilities.

Strike price	$80.00
Days to expiration	15
Standard deviation	45.0%
CC Risk-free rate	5.0%
Dividends	$0.00

Black-Scholes value of an option equals the cost of setting up a self-financing hedge.
Setup costs = (Option delta X stock price) - present value of(probability finish in the money X strike price).

Assets					**Liabilities**				
Days to Exp	Stock Price	Option Delta	Number of Shares Required for Delta Hedge—Delta X Number of Options	**Cost of Shares Purchased—Number of Shares X Stock Price**	Probability Option will Finish in the Money	Value of Bond at Expiry—Probability In-Money X Strike Price	**Hedge Require-ment—Bond Value Today**	**Value of Options—Cost of Shares Purchased - Bond Value Today**	Black-Scholes Option Value
15	$78.25	0.43081	4,308	**$337,107**	0.39530	$316,237	$315,588	**$21,520**	$2.15197

In theory, once a trader sets up a delta hedge, it doesn't cost him any additional money to maintain it

In the theory that underlies the Black-Scholes Option Pricing Model, a trader rebalances his hedge continuously. Assets and liabilities stay perfectly in balance.

Once a trader sets up a delta hedge, any additional shares he needs to maintain the delta hedge he buys with money he borrows. In theory, to keep the hedge balanced, he requires no additional infusion of funds. The cost of a delta hedge is the cost of setting it up. Hence, an option's Black-Scholes value is the cost to a trader of setting up a delta hedge.

In our example earlier, we saw that at the set up of the delta hedge, the assets equal the liabilities.

Assets	
Market value of delta shares of stock	$337,107.20
Liabilities	
Present value of bond obligation	$315,587.50
Value of options sold	21,519.70
Total liabilities	$337,107.20

We did not say how the option value was derived.

The Black-Scholes value of an option is the cost of setting up the hedge. It is the difference between the value of delta shares of stock and the value of the bond.

	Market value of delta shares of stock	$337,107.20
-	Present value of bond obligation	315,587.50
	Value of 10,000 options sold	$ 21,519.70
	Value of one option	$2.15197

The table above shows the cost of setting up the hedge in our example.

	Annualized	Period
Standard Deviation	45.00	9.1225%
No Dividend	0.00	0.00
CC Risk-free Rate	5.00	0.2055%

Current Asset Price	78.25
Days to Expire (365 D/Y)	15
Call Strike Price	80.00
Black-Scholes Value	2.15

Calculate Black-Scholes Value

To calculate the Black-Scholes value of an option, the animation calculates the cost of setting up the delta hedge

To calculate the Black-Scholes value of a call, the animation goes through the same steps shown in the spreadsheet examples.

1. Click three times on Clear.
2. Click twice on Hide.
3. Click on Calculate Black-Scholes Value.
4. For Standard Deviation, enter 45.00.
5. For CC Risk-free Rate, enter 5.0.
6. For Current Asset Price, enter 78.25.
7. For Days to Expire, enter 15.
8. For Call Strike Price, enter 80.00.
9. Click again on Calculate Black-Scholes Value.

Option delta	0.43081
X Current asset price	$78.25
= Cost of delta shares	$33.71072
Probability finish in-money	0.39530
X Strike price	$80.00
= Bond value at expiration	$-31.62367
Bond value at expiration	$-31.62367
÷ Exp(period risk-free rate)	1.00206
= Bond value today	$-31.55875
Cost of delta shares	$33.71072
+ Bond value Today	$-31.55875
= Black-Scholes value	$ 2.15197

Black-Scholes Assumptions (Part II)

Delta hedging brings into play several additional Black-Scholes assumptions

Earlier we reviewed some of the assumptions that underlie Black-Scholes Options-Pricing Theory:

1. Stock returns expressed as geometric rates of return are normally distributed.
2. Price changes are lognormally distributed.
3. The potential price paths of a stock can be characterized by a geometric Brownian motion model.
4. The volatility of a stock's price path is constant over the investment horizon.

Delta hedging brings into play several additional assumptions:

5. Stock-price paths are continuous. There are no discontinuous jumps in price changes.
6. A trader or investor can trade continuously.
7. The financial markets are perfectly liquid. Hence, a trader's attempt to buy or sell will not move prices up or down.
8. If he wishes to make a risk-free investment, a trader or investor can borrow at the risk-free rate.
9. A trader or investor can sell stock short and have full use of the proceeds.
10. The risk-free rate of interest will not change over the investment horizon.
11. Traders and investors incur no transaction costs such as bid-ask spreads or brokerage fees and commissions.
12. The price of an option is the cost of hedging a position in that option.
13. The financial markets do not permit the existence of arbitrage opportunities.
14. Investors are risk neutral. That is, they do not require greater expected returns as compensation for taking on greater exposures to uncertainty.

Number of options	10,000		Standard deviation	45.0%
Strike price	$80.00		CC Risk-free rate	5.0%
Days to expiration	15		Dividends	$0.00

Daily Delta Hedging in Practice—Borrowing Enough Money to Cover Stock Purchases

Liabilities										
Days to Exp	Stock Price	Value of Option	Probability Option will Finish in the Money	Value of Bond at Expiry—Probability In-Money X Strike Price	Hedge Require-ment—Bond Value Today	One Day Interest on Previous Day's Total Bond	Money Borrowed for or Repaid from Stock Transaction	Total Bond Obligation—Borrowing + Interest	Value of Options Sold	**Total Liabilities—Borrowings + Interest due + Market Value of Options sold**
15	$78.25	$2.152	0.39530	$316,237	$315,588	$0.00	$315,588	$315,588	$21,520	**$337,107**
14	$82.52	$4.387				$43.23	$190,765	$506,396	$43,868	**$550,264**
13	$81.74	$3.780				$69.37	-$30,689	$475,776	$37,801	**$513,577**
12	$79.51	$2.418				$65.18	-$103,322	$372,519	$24,179	**$396,698**
11	$78.67	$1.912				$51.03	-$44,280	$328,291	$19,122	**$347,413**
10	$80.28	$2.579				$44.97	$82,669	$411,005	$25,790	**$436,795**
9	$82.25	$3.649				$56.31	$107,894	$518,956	$36,486	**$555,442**
8	$80.00	$2.169				$71.09	-$121,617	$397,410	$21,688	**$419,097**
7	$77.36	$0.935				$54.44	-$161,210	$236,254	$9,346	**$245,600**
6	$77.63	$0.889				$32.37	$3,836	$240,123	$8,890	**$249,013**
5	$78.14	$0.912				$32.90	$20,045	$260,200	$9,117	**$269,317**
4	$77.89	$0.673				$35.65	-$34,703	$225,532	$6,733	**$232,265**
3	$79.29	$0.985				$30.90	$101,625	$327,188	$9,854	**$337,042**
2	$80.08	$1.115				$44.82	$77,298	$404,531	$11,152	**$415,683**
1	$82.14	$2.275				$55.42	$288,055	$692,641	$22,752	**$715,394**
0	$83.06	$3.060				$94.89	$105,988	$798,724	$30,600	**$829,324**

Delta hedging in practice is different from delta hedging in theory

In actuality, the examples in the spreadsheet tables did not satisfy the Black-Scholes assumptions. We did not rebalance the hedge continuously. We didn't even rebalance it whenever there was a small change in delta. Instead, we rebalanced once a day. With daily rebalancing, the amount of money borrowed was not always sufficient to cover the cost of the shares purchased.

In practice, a trader simply may borrow the amount of money necessary to cover the purchase of the shares required to maintain the delta hedge. The table above shows this practice.


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$31.81
$28.79
$26.05
$23.57
End-of-period Asset Price $57.24
- Strike Price 80.00
= Payoff $0
Payoff $0
x Probability 0.00059172
= Probability-weighted Future Value 0.00000000
Probability-weighted FV / 0.00000000
(exp(Period Expected R)) 1.00494369
= Probability-weighted Present Value 0.00000000
Cumulative Probability-weighted Net Present Value 2.2432243
Annualized Period
Expected CC Return 12.00 0.4932%
Median CC Return 1.875% 0.0771%
Standard Deviation 45.00 9.1225%
No Dividend 0.00 0.00
CC Risk-free Rate 5.00 0.2055%
Current Asset Price 78.25
Days to Expire (365 D/Y) 15
Call Strike Price 80.00
Black-Scholes Value 2.15
Calculate Black-Scholes Value
Calculate Option Value
Fast
Fastest

The Black-Scholes assumptions envision a risk-neutral world—a world in which every asset's expected return is equal to the risk-free rate of return

Earlier, to calculate an option's probability-weighted net present value, we used a forecast of the stock's expected return.

1. Click three times on Clear.
2. Click two or three times on Hide.
3. Click on Calculate Option Value.
4. For Expected CC Return, enter 12.00.
5. For Standard Deviation, enter 45.00.
6. For Current Asset Price, enter 78.25.
7. For Days to Expire, enter 15.
8. For Call Strike Price, enter 80.00.
9. Click on Fastest to the right of Calculate Option Value.

In our example, for an expected return of 12%, we got a probability-weighted net present value of $2.24.

When we calculated the Black-Scholes value of the option, we got a value of $2.15.

10. Click on Calculate Black-Scholes Value.
11. For CC Risk-free Rate, enter 5.00.
12. Click on Calculate Black-Scholes Value.

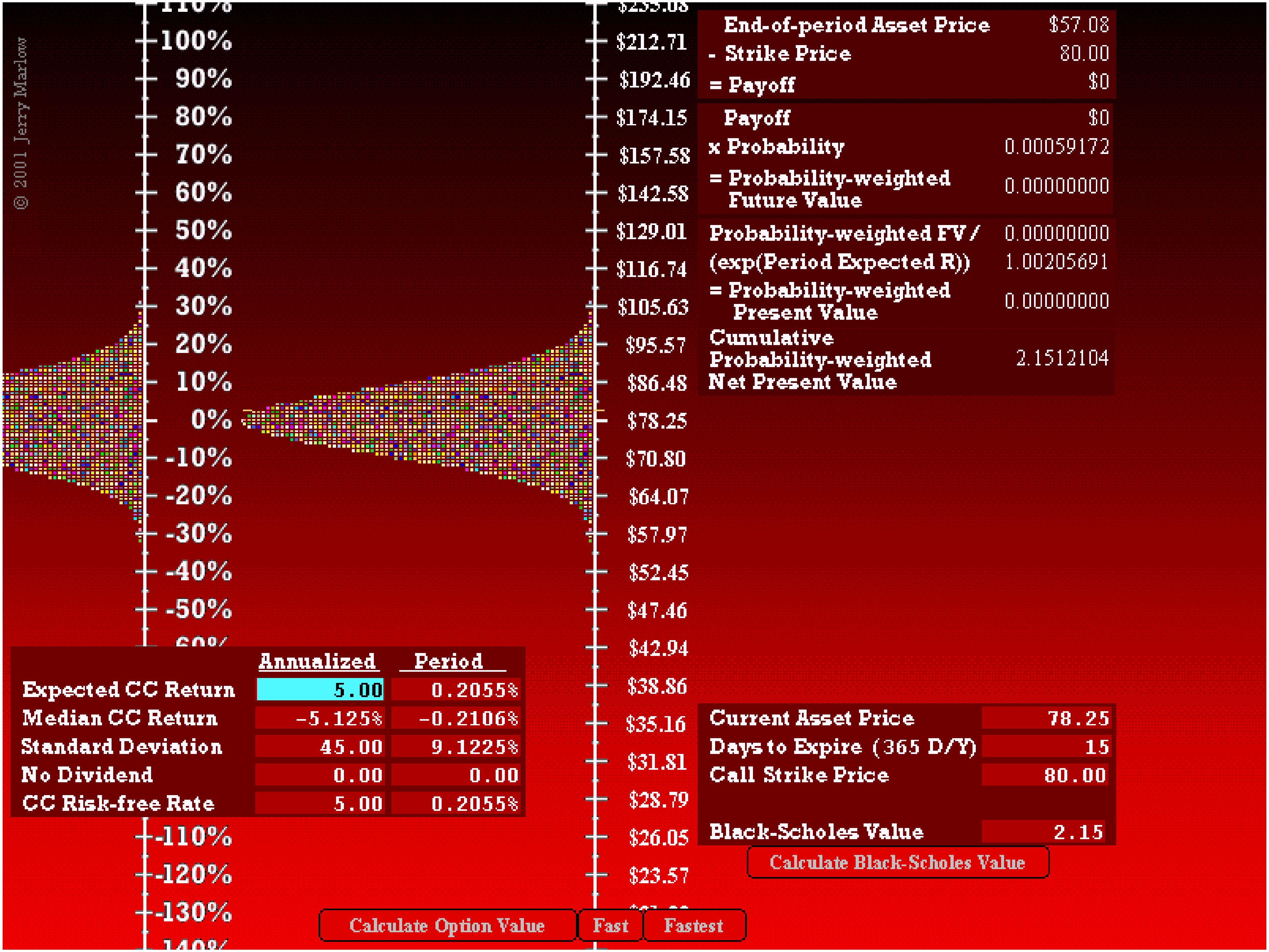
If you were willing to pay $2.24 for this option, a trader could hedge the sale for $2.15 and turn a risk-free profit. He could perform arbitrage.

Why the difference in the two valuations?

The Black-Scholes value is the cost of hedging the sale of the option. Nowhere in the Black-Scholes formulas does the expected return of the underlying stock appear.

The Black-Scholes formulas assume that the financial markets do not allow arbitrage opportunities to exist. They assume or imply that investors do not require higher expected returns for taking on an exposure to uncertainty. The formulas assume or imply a risk-neutral world—a world in which every asset's average return is equal to the risk-free rate of return.

As we've seen earlier, an investment's expected return is the average of all the returns in its probability distribution. In the risk-neutral world of Black-Scholes, the expected return of every investment is the risk-free rate.



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$31.81
$28.79
$26.05
$23.57
End-of-period Asset Price $57.08
- Strike Price 80.00
= Payoff $0
Payoff $0
x Probability 0.00059172
= Probability-weighted Future Value 0.00000000
Probability-weighted FV / 0.00000000
(exp(Period Expected R)) 1.00205691
= Probability-weighted Present Value 0.00000000
Cumulative Probability-weighted Net Present Value 2.1512104
Annualized
Period
Expected CC Return 5.00 0.2055%
Median CC Return -5.125% -0.2106%
Standard Deviation 45.00 9.1225%
No Dividend 0.00 0.00
CC Risk-free Rate 5.00 0.2055%
Current Asset Price 78.25
Days to Expire (365 D/Y) 15
Call Strike Price 80.00
Black-Scholes Value 2.15
Calculate Black-Scholes Value
Calculate Option Value
Fast
Fastest

To bring an option's probability-weighted net present value into line with its Black-Scholes value, set the stock's expected return equal to the risk-free rate

If you accept Black-Scholes's no-arbitrage, risk-neutral assumptions, then you can bring an option's probability-weighted net present value into line with its Black-Scholes value. All you have to do is set the expected return of your forecast equal to the risk-free rate.

13. Click on Clear.
14. For Expected CC Return, enter the risk-free rate of 5.0.
15. Click on Calculate Option Value Fastest.

With the expected return equal to the risk-free rate, the option's probability-weighted net present value is equal to or extremely close to the Black-Scholes value.

In the opinion of many, the great brilliance of Fischer Black and Myron Scholes was not that they figured out how to value stock options. We have seen that we can find the same value by setting the expected return equal to the risk-free rate and calculating the option's probability-weighted net present value. As my Uncle Glenn used to say at the dinner table whenever his son-in-law Charles would give voice to his insights into life's most profound mysteries, "Hell! Ev'r'body knows that!"

The great brilliance of Black and Scholes was that they figured out how a market-maker can use delta hedging to sell options risk free.

Is it coincidence that—when you use the same expected returns—an option's Black-Scholes value and its probability-weighted net present value are the same? Or do the laws of markets and mathematics require that they be the same? On your next Zen retreat, let that question be your koan.


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$74.59
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$61.07
$55.26
$50.00
$45.24
$40.94
$37.04
$33.52
$30.33
$27.44
$24.83
$22.47
$20.33
$18.39
$16.64
$15.06
End-of-period Asset Price $0.74
- Strike Price 500.00
= Payoff $0
Payoff $0
x Probability 0.00059172
= Probability-weighted Future Value 0.00000000
Probability-weighted FV / 0.00000000
(exp(Period Expected R)) 1.06183655
= Probability-weighted Present Value 0.00000000
Cumulative Probability-weighted Net Present Value 0.9057153
Animation: 1,690 intervals: 0.9057153
Ctrl+A: 10,000 intervals: 0.9781469
Ctrl+B: 100,000 intervals: 0.9974290
Ctrl+C: 1,000,000 intervals: 1.0004003
Annualized Period
Expected C Return 6.00 6.0000%
Median C Return -51.986% -51.9859%
Standard eviation 107.6902 107.6902%
No Divid d 0.00 0.00
CC Risk- e Rate 6.00 6.0000%
Current Asset Price 50.00
Days to Expire (365 D/Y) 365
Call Strike Price 500.00
Black-Scholes Value 1.00
Calculate Black-Scholes Value
-110%
-120%
-130%
Calculate Option Value
Fast
Fastest

For strike prices at extremes of widely-spread probability distributions, Monte Carlo simulations do not calculate option values very accurately. Use Ctrl+A, B, or C.

Earlier we saw that, when a strike price is at an extreme of a stock's probability distribution, Monte Carlo simulations do not calculate the option's expected return very accurately. It has too few in-the-money outcomes to work with. To improve the accuracy, we increased the number of intervals into which the simulation divided the probability distribution.

Similarly, when a strike price is at either extreme of a widely-spread probability distribution, Monte Carlo simulations do not calculate very accurately an option's probability-weighted net present value.

For a probability distribution with a large standard deviation, let's compare the Black-Scholes value and the probability-weighted net present value when expected return and risk-free rate are the same. The values should be the same.

16. Click three times on Clear.
17. For Expected CC Return, enter 6.00.
18. For Standard Deviation, enter 107.6902.
19. For CC Risk-free Rate, enter 6.00.
20. For Current Asset Price, enter 50.00.
21. For Days to Expire, enter 365.
22. For Call Strike Price, enter 500.00.
23. Click on Calculate Black-Scholes value.

The Black-Scholes value is $1.00.

24. Click on Calculate Option Value Fast or Fastest.

The animation calculates the probability-weighted net present value to be 0.9057153 or roughly $0.91. Not very close.

To improve the accuracy, we can increase the number of intervals into which the simulation divides the probability distribution.

25. To have the Monte Carlo simulation divide the probability distribution into 10,000 intervals, hold down the Ctrl key on your keyboard and press the A key.

The routine calculates the probability-weighted net present value to be 0.9781469. Closer, but still not the right answer.

26. To divide the probability distribution into 100,000 intervals, hold down the Ctrl key and press the B key.

The routine calculates the probability-weighted net present value to be 0.9974290. Very close. To the nearest penny, it would round up to $1.00.

27. To divide the probability distribution into 1,000,000 intervals, hold down the Ctrl key and press the C key.
28. Go have a latte.

By dividing the probability distribution into a million intervals, the Monte Carlo routine calculates the probability-weighted net present value to be 1.0004003.

29. If a calculation or animation is taking too long, to interrupt it, click long and hard on any of the empty areas on the screen.

The Black-Scholes value of a put is the amount of money a trader has to charge for it to set up a delta hedge with her assets and liabilities in balance

If a put finishes in the money, the trader has to buy from the option holder the underlying at the strike price. To be perfectly hedged for this outcome, at the time of expiration the trader wants to be short all the shares she has to buy.
To pay for the stock she has to buy, she wants to liquidate a loan she has made.

If a put finishes out of the money, the holder of the option will not exercise it. For this outcome, the trader does not want to be short any stock. She does not want to be owed any money.

To set up a hedge for the sale of a put, a trader sells short some of the stock. The proceeds of the short sale and the money she receives for the option the trader lends at the risk-free rate.

The amount of stock the trader sells short is determined by the option's delta. The delta of a put is negative. As the value of the underlying stock goes up, the value of the put goes down. If we preserve the minus sign in front of the delta, we can say—as with setting up a hedge for a call option—that the trader "buys" delta shares of stock. A negative "buy" is a sell.

For example:

	Option delta	-0.56919
X	Current asset price	$78.25000
=	Cost of delta shares	$-44.53928

A negative "cost" means the trader receives money.

The amount of money the trader lends out is determined by the probability that the option will finish in the money and the strike price. The trader lends out the present value of the probability the option will finish in the money times the strike price.

	Probability finish in the money	0.60470
X	Strike price	$80.00000
=	Bond value at expiration	$48.376333
	Bond value at expiration	$48.37633
÷	exp(period risk-free rate)	1.00206
=	Bond value today	$48.27703

When the trader sets up the hedge, she wants her assets and liabilities to be in balance. The missing number is the amount of money she charges for the option. The Black-Scholes value of the option is the amount of money she has to charge for it for the delta-hedge assets and liabilities to be in balance.

	Cost of delta shares	$-44.53928
+	Bond value today	$48.27703
	Black-Scholes value	$3.73775

Upon setting up the hedge, the hedge's assets and liabilities look like this:

Assets	
Bond value	$48.27703
Liabilities	
Value of delta shares	$44.53928
Value of options sold	$ 3.73775
Total liabilities	$48.27703

	Annualized	Period
Standard Deviation	45.00	9.1225%
No Dividend	0.00	0.00
CC Risk-free Rate	5.00	0.2055%

Current Asset Price	78.25
Days to Expire (365 D/Y)	15
Put Strike Price	80.00
Put Option Price	
Black-Scholes Value	3.74

Put

Calculate Black-Scholes Value

To calculate the Black-Scholes value of a put, the animation calculates the cost of setting up a delta hedge

1. Click three times on Clear.
2. Click twice on Hide.
3. Click on Calculate Black-Scholes Value.
4. Click on Put.
5. For Standard Deviation, enter 45.00.
6. For CC Risk-free Rate, enter 5.0.
7. For Current Asset Price, enter 78.25.
8. For Days to Expire, enter 15.
9. For Put Strike Price, enter 80.00.
10. Click again on Calculate Black-Scholes Value.

The Black-Scholes value of a put is

	Option delta	-0.56919
X	Current asset price	$78.25000
=	Cost of Delta Shares	$-44.53928
	Probability finish in the money	0.60470
X	Strike price	$80.00000
=	Bond value at expiration	$48.376333
	Bond value at expiration	$48.37633
÷	exp(period risk-free rate)	1.00206
=	Bond value today	$48.27703
	Cost of delta shares	$-44.53928
+	Bond value today	$48.27703
	Black-Scholes value	$3.73775

What to remember about hedging: The value of an option to you is its probability-weighted net present value. The price of an option is the cost to a market maker of hedging the sale plus a profit margin.

If you plan on making a market in options and selling them to investors, you'll need the ability to hedge complex positions in many different options with different expiration dates and strike prices. To hedge those complex positions, you'll need a more thorough understanding of hedging than is offered here. You'll need software with a different focus than *Black-Scholes Made Easy*.

The intention here is to show that the Black-Scholes value of an option is the cost to the trader or market maker of hedging the sale of the option. If we apply the Black-Scholes assumptions to the calculations, then an option's probability-weighted net present value is equal to or very nearly equal to the Black-Scholes value.

These are principles that apply generally in the financial markets. The value of an asset is its probability-weighted net present value. The price of synthetic financial instruments like options and other derivatives is the cost to the market maker of hedging the sale plus a profit margin.

To price synthetic financial instruments, financial institutions often use Monte Carlo simulations to calculate their probability-weighted net present values. To avoid risk in their sales of synthetic instruments, financial institutions hedge their positions. They make money on bid/ask spreads and on transaction fees and commissions. If they stay perfectly hedged, they earn profits whether the value of an asset goes up, down, or remains the same.

If you buy derivatives and you don't comprehend the pricing methodology and know the forecasts for the underlyings, then you don't know how big a profit margin you're forking over. You may be the one who gets forked.

When and why you can gain an advantage from early exercise of some American options and Black's approximation for valuing American options

Under some circumstances, you can gain an advantage if you exercise an American option prior to maturity. The right of early exercise gives some American options greater value than otherwise identical European options. Accordingly, they command a higher price in the marketplace.

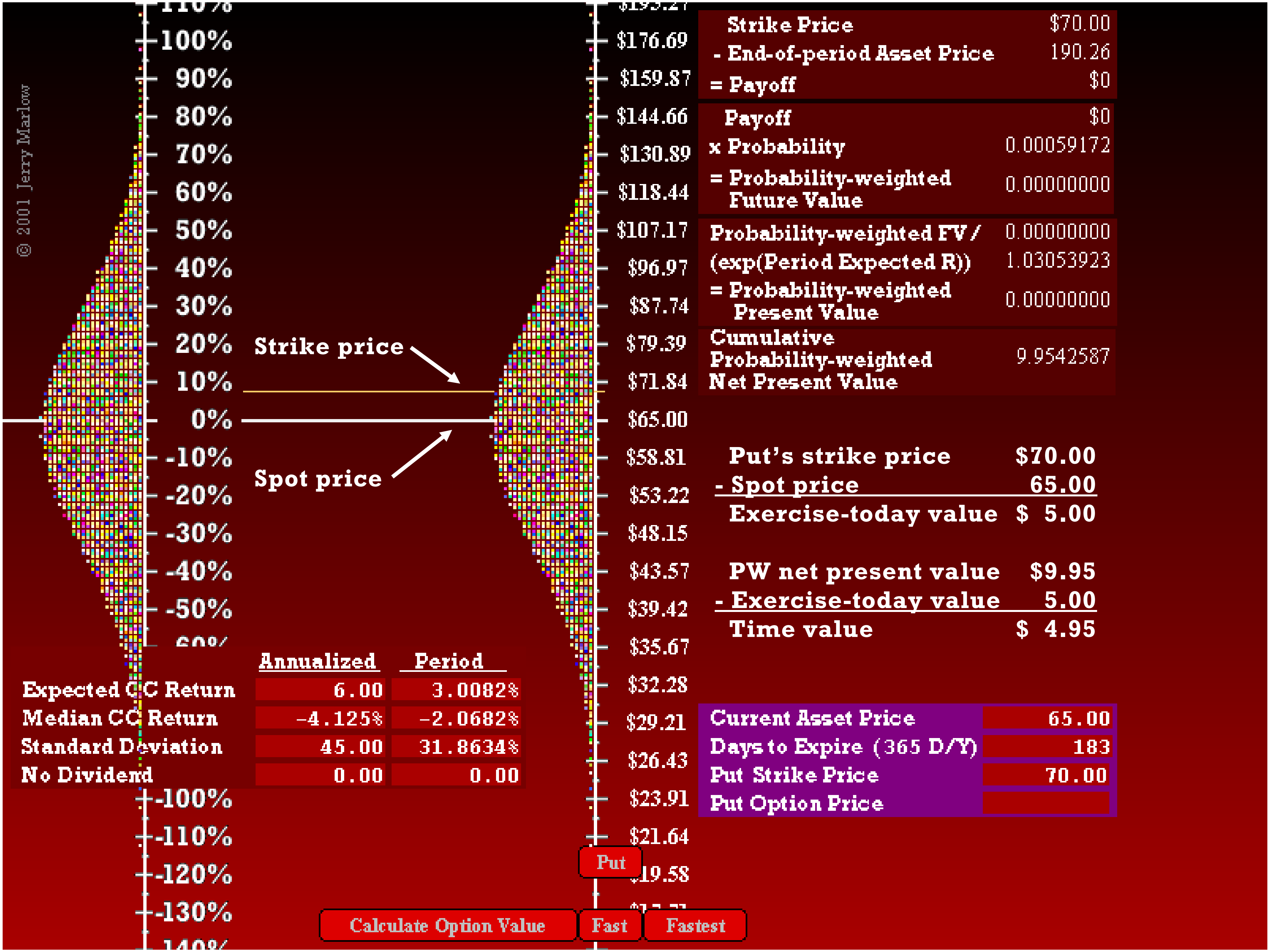
Up until now, we've been looking at potential outcomes of investing in options and holding them until maturity. Implicitly we've been assuming that all options are European options—that they can be exercised only at maturity.

Most stock options, however, are American. They give you all the rights that European options give plus the right of early exercise.

Under certain circumstances, exercising some types of options prior to maturity gives you an advantage. The probability-weighted net present value and expected return of early or immediate exercise is greater than that of later exercise.

Insofar as early exercise gives you a potential advantage, American options have greater value than otherwise identical European options. In the marketplace, they command a higher price.

Now we look at the types of options and the circumstances under which early exercise can give you an advantage. First we establish criteria for how we tell whether early exercise is advantageous.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Strike price
Spot price
$176.69
$159.87
$144.66
$130.89
$118.44
$107.17
$96.97
$87.74
$79.39
$71.84
$65.00
$58.81
$53.22
$48.15
$43.57
$39.42
$35.67
$32.28
$29.21
$26.43
$23.91
$21.64
$19.58
Strike Price $70.00
- End-of-period Asset Price 190.26
= Payoff $0
Payoff $0
x Probability 0.00059172
= Probability-weighted Future Value 0.00000000
Probability-weighted FV / 0.00000000
(exp(Period Expected R)) 1.03053923
= Probability-weighted Present Value 0.00000000
Cumulative Probability-weighted Net Present Value 9.9542587
Put's strike price $70.00
- Spot price 65.00
Exercise-today value $ 5.00
PW net present value $9.95
- Exercise-today value 5.00
Time value $ 4.95
Annualized Period
Expected CC Return 6.00 3.0082%
Median CC Return -4.125% -2.0682%
Standard Deviation 45.00 31.8634%
No Dividend 0.00 0.00
Current Asset Price 65.00
Days to Expire (365 D/Y) 183
Put Strike Price 70.00
Put Option Price
Put
Calculate Option Value
Fast
Fastest

If an option's probability-weighted net present value of future exercise is greater than its exercise-today value, the option has time value. It makes sense to hold on to it.

One of the ways that we've been using to find the value today of an option that we plan to hold to maturity has been to sweep through the underlying's probability distribution of possible end-of-period outcomes, from the outcomes compute the option payoffs, and discount the payoffs to an interest-adjusted value for today. We call this adjusted value the option's probability-weighted net present value of end-of-period exercise.

1. Click three times on Clear.
2. Click twice on Hide.
3. Click on Put.
4. Click on Calculate Option Value.
5. For Expected CC Return, enter 6.00.
6. For Standard Deviation, enter 45.00.
7. For Current Asset Price, enter 65.00.
8. For Days to Expire, enter 183.
9. For Put Strike Price, enter 70.00.
10. Click on Calculate Option Value Fast or Fastest. (You can jump back and forth.)

In working with options, it's easy to get pulled out of today and start thinking about what might happen some time in the future. Instead of trying to work with hopes and fears that tend to run wild, we can work with probability-weighted net present values.

Probability-weighted net present values are not future values. They do not in any way depend on what you might have paid for an option. They are values today of all those outcomes you think possibly might happen in the future. They are values we can compare and use to make decisions today.

In our example, rounded to the nearest penny, the option's probability-weighted net present value is $9.95.

The easiest number to compare to an option's probability-weighted net present value is its exercise-today value—the payoff you would get if you exercised the option today. (You even can think of the exercise-today value as a probability-weighted net present value. There's only one outcome. It's 100% certain. The time of discount is zero.)

A put's exercise-today value is the difference between the option's strike price and the current market price or spot price of the underlying.

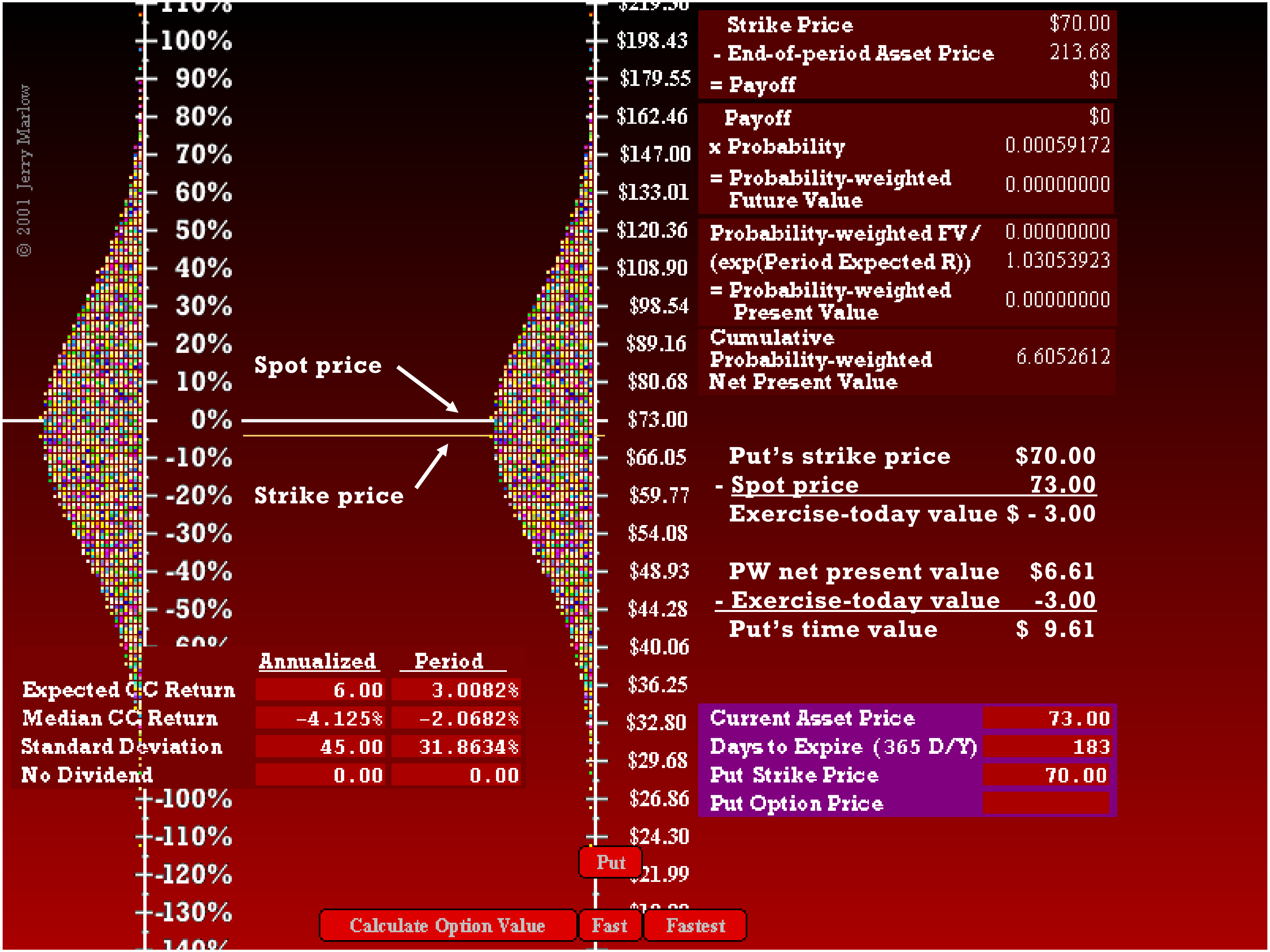
In our example, the option is in the money. Its exercise-today value is $70.00 - $65.00 = $5.00.

The probability-weighted net present value of holding the option until maturity is greater than the exercise-today value. It's $4.95 greater.

When the probability-weighted net present value of holding onto an option is greater than its exercise-today value, we say that the option has time value. On average, you expect it to have a greater exercise payoff in the future than it has today.

An option's time value is the difference between its probability-weighted net present value of future exercise and its exercise-today value. Here the option's time value is
$9.95 – $5.00 = $4.95

If you owned this option, it would not make sense to exercise it today. You would lose value. You would lose the option's time value.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Spot price
Strike price
$198.43
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$120.36
$108.90
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$89.16
$80.68
$73.00
$66.05
$59.77
$54.08
$48.93
$44.28
$40.06
$36.25
$32.80
$29.68
$26.86
$24.30
$21.99
Strike Price $70.00
- End-of-period Asset Price 213.68
= Payoff $0
Payoff $0
x Probability 0.00059172
= Probability-weighted Future Value 0.00000000
Probability-weighted FV / 0.00000000
(exp(Period Expected R)) 1.03053923
= Probability-weighted Present Value 0.00000000
Cumulative Probability-weighted Net Present Value 6.6052612
Put's strike price $70.00
- Spot price 73.00
Exercise-today value $ - 3.00
PW net present value $6.61
- Exercise-today value -3.00
Put's time value $ 9.61
Annualized
Period
Expected CC Return 6.00 3.0082%
Median CC Return -4.125% -2.0682%
Standard Deviation 45.00 31.8634%
No Dividend 0.00 0.00
Current Asset Price 73.00
Days to Expire (365 D/Y) 183
Put Strike Price 70.00
Put Option Price
Put
Calculate Option Value
Fast
Fastest

If an option is out of the money, it has no exercise-today value. It has only time value. It would make no sense to exercise it today.

Now let's compare probability-weighted net present value of end-of-period exercise, exercise-today value, and time value for a put that is out of the money.

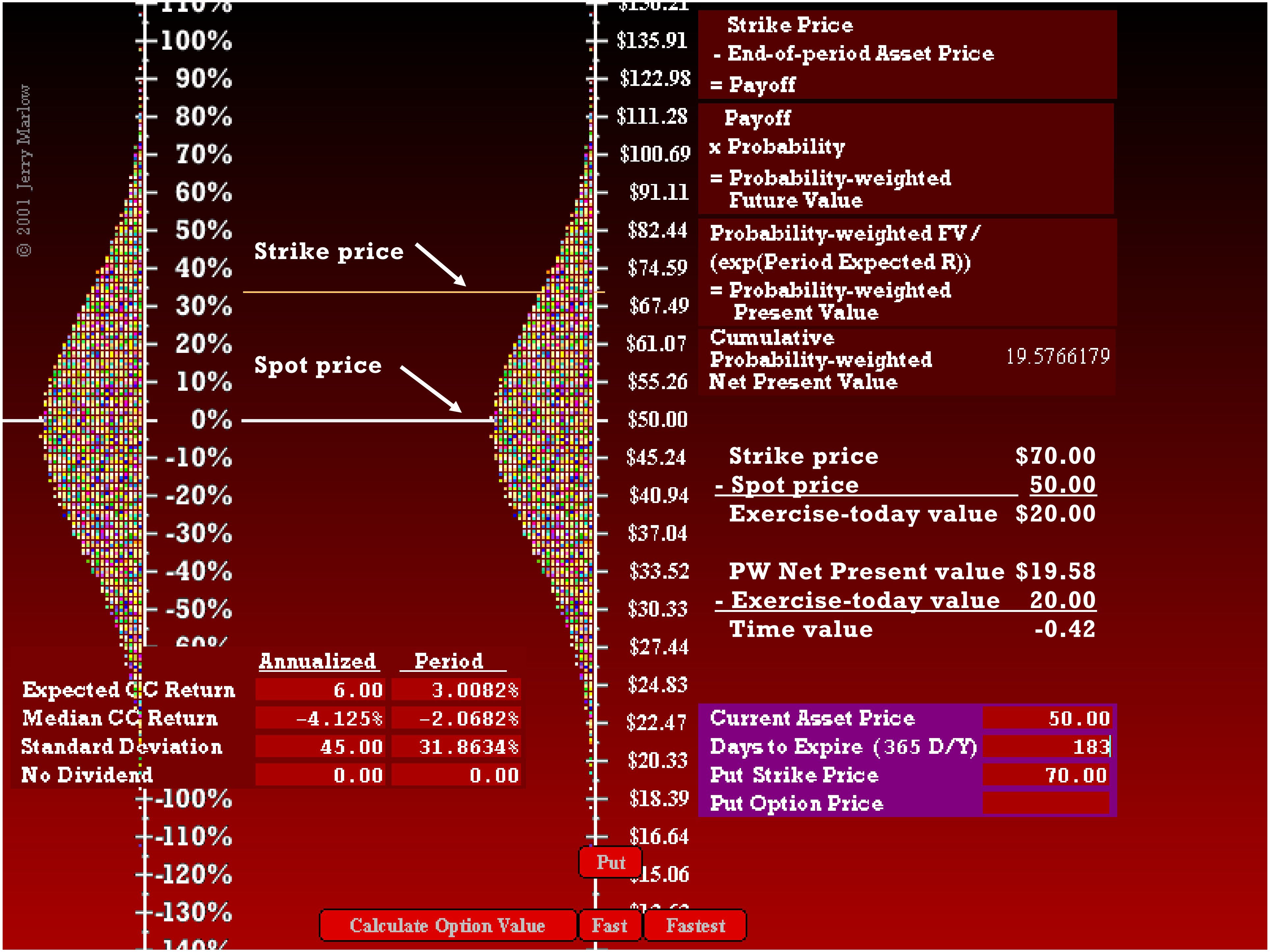
11. For Current Asset Price, enter 73.00.
12. Click on Calculate Option Value Fast or Fastest.

Rounded to the nearest penny, the option's probability-weighted net present value of end-of-period exercise is $6.61.

The spot price is greater than the strike price. Hence, the put is out of the money. Its exercise-today value is zero.

If you exercised the put today, you would compel someone to buy from you for $70 a stock with a market value of $73. You would lose money. You would lose $3.

The option's only value is its time value—the difference between its probability-weighted net present value and its exercise-today value. In this example, the time value is $6.61 minus -$3.00 = $9.61. You definitely would not want to exercise this option today.



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Strike price
Spot price
$135.91
$122.98
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$100.69
$91.11
$82.44
$74.59
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$61.07
$55.26
$50.00
$45.24
$40.94
$37.04
$33.52
$30.33
$27.44
$24.83
$22.47
$20.33
$18.39
$16.64
$15.06
Strike Price
- End-of-period Asset Price
= Payoff
Payoff
x Probability
= Probability-weighted Future Value
Probability-weighted FV / (exp(Period Expected R))
= Probability-weighted Present Value
Cumulative Probability-weighted Net Present Value 19.5766179
Strike price $70.00
- Spot price 50.00
Exercise-today value $20.00
PW Net Present value $19.58
- Exercise-today value 20.00
Time value -0.42
Annualized
Period
Expected CC Return 6.00 3.0082%
Median CC Return -4.125% -2.0682%
Standard Deviation 45.00 31.8634%
No Dividend 0.00 0.00
Current Asset Price 50.00
Days to Expire (365 D/Y) 183
Put Strike Price 70.00
Put Option Price
Put
Calculate Option Value
Fast
Fastest

As a put goes deep into the money, it may be advantageous to exercise the option as soon as its exercise-today value becomes equal to the probability-weighted net present value of holding on to it until maturity.

Now let's compare values for a put that has gone deep in the money.

13. For Current Asset Price, enter 50.00.
14. Click on Calculate Option Value Fast or Fastest.

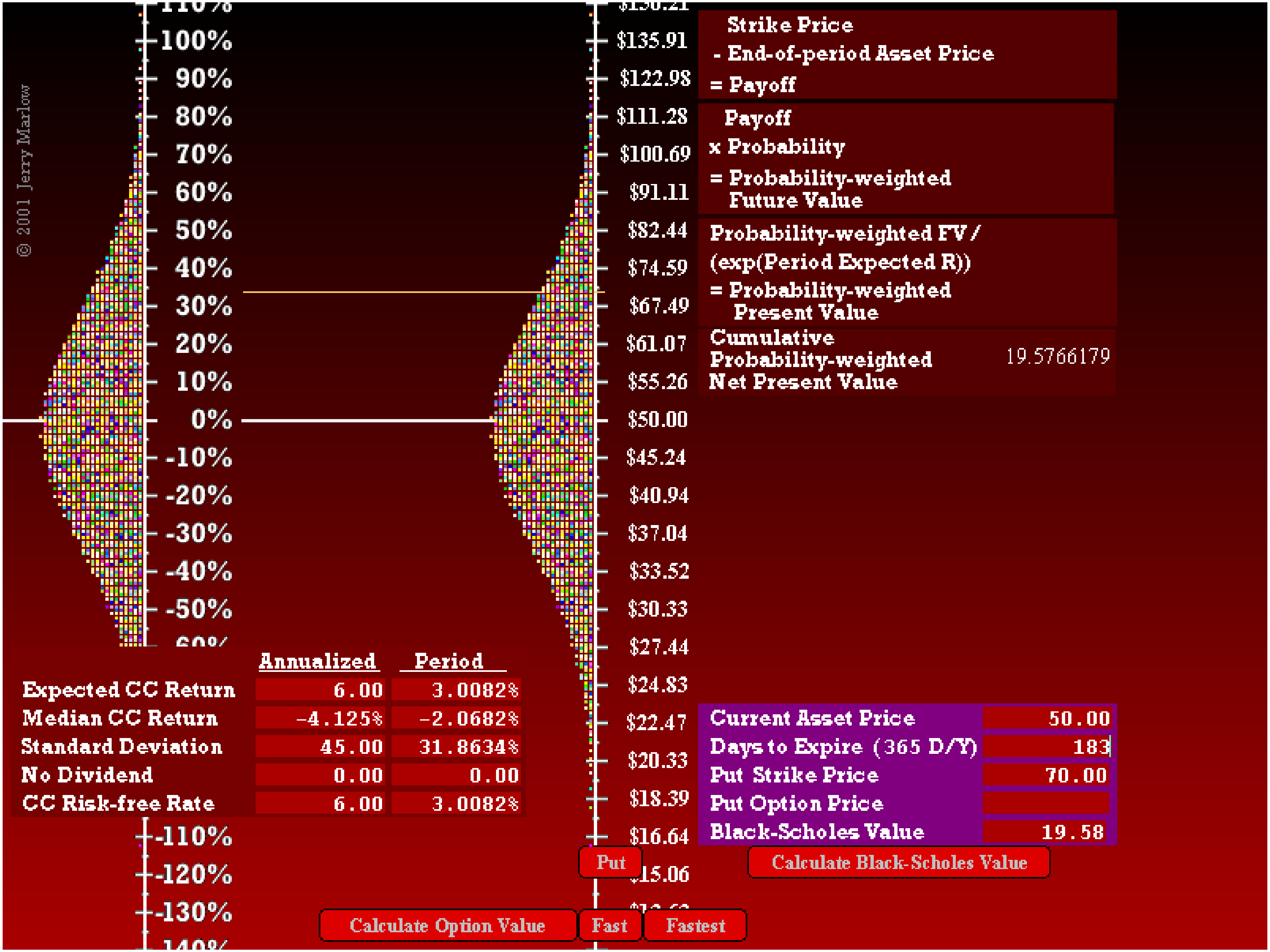
Rounded to the nearest penny, the option's probability-weighted net present value of end-of-period exercise is $19.58.

The option's exercise-today value is $70.00 - $50.00 = $20.00.

The option's exercise-today value is greater than its probability-weighted net present value of end-of-period exercise. It's $0.42 greater.

The option has negative time value. On average, you can expect the option to have a lower exercise payoff in the future than it has today.

If you owned this option, the logical thing to do would be to exercise it today. In fact—except under weird assumptions—it becomes logical to exercise an option as soon as its exercise-today value becomes equal to the probability-weighted net present value of future exercise.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$45.24
$40.94
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$30.33
$27.44
$24.83
$22.47
$20.33
$18.39
$16.64
Strike Price
- End-of-period Asset Price
= Payoff
Payoff
x Probability
= Probability-weighted Future Value
Probability-weighted FV / (exp(Period Expected R))
= Probability-weighted Present Value
Cumulative Probability-weighted Net Present Value
19.5766179
Annualized
Period
Expected CC Return 6.00 3.0082%
Median CC Return -4.125% -2.0682%
Standard Deviation 45.00 31.8634%
No Dividend 0.00 0.00
CC Risk-free Rate 6.00 3.0082%
-110%
-120%
-130%
Current Asset Price 50.00
Days to Expire (365 D/Y) 183
Put Strike Price 70.00
Put Option Price
Black-Scholes Value 19.58
Put
Calculate Black-Scholes Value
Calculate Option Value
Fast
Fastest

An option's value is the greater of its Black-Scholes value and its early-exercise value—Black's approximation

What is the value of this option?

We calculated its probability-weighted net present value of end-of-period exercise to be $19.58. Earlier we saw that, when an option's expected return is equal to the risk-free rate, its Black-Scholes value is equal to its probability-weighted net present value of end-of-period exercise.

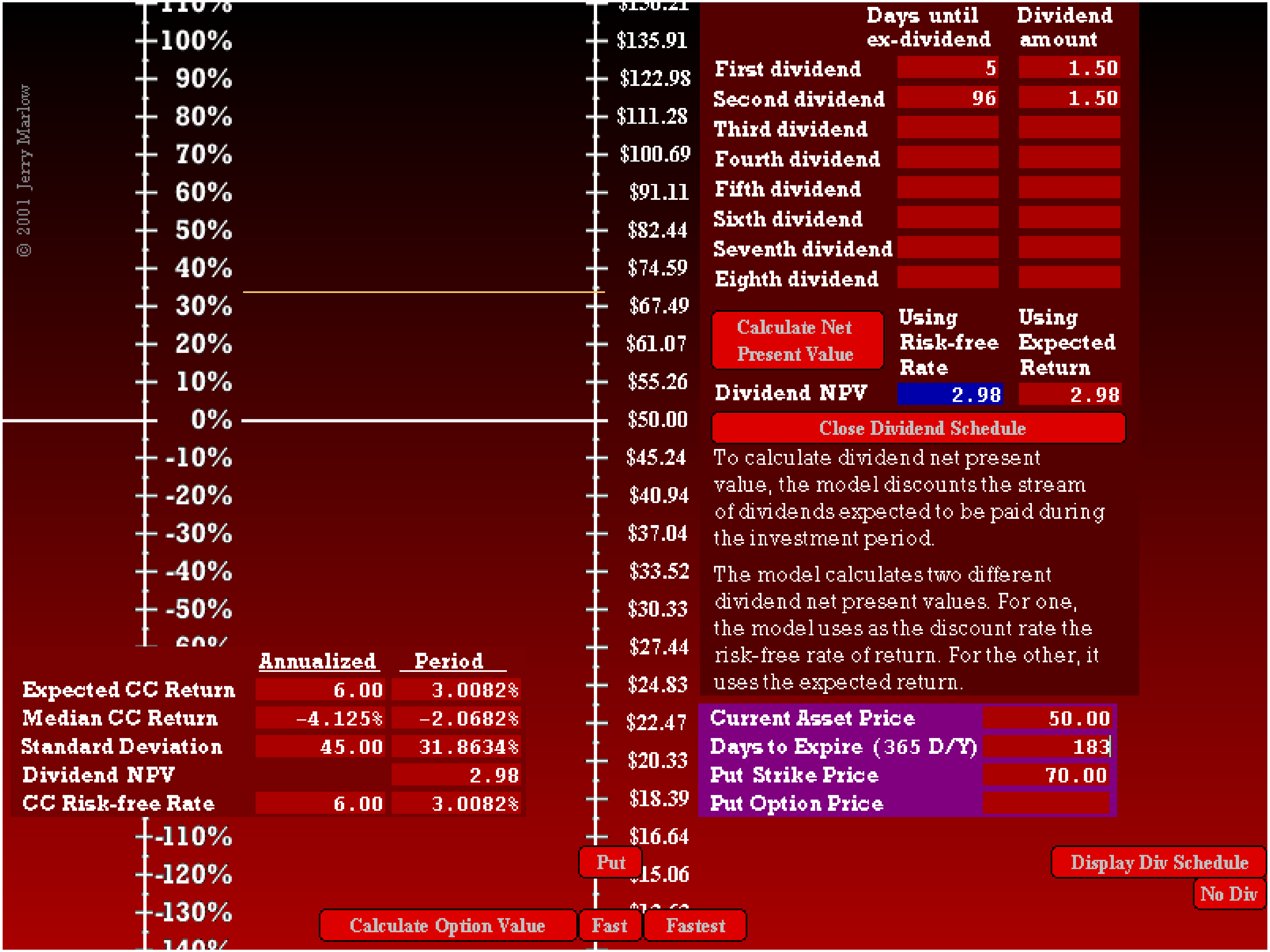
15. Click on Calculate Black-Scholes Value.
16. For CC Risk-free rate, enter 6.00.
17. Click on Calculate Black-Scholes Value.

We see that, with a risk-free rate of 6%, the option's Black-Scholes value is also $19.58.

Clearly the market price of the option cannot be $19.58. If it was, it would be a perpetual-money machine. You could keep buying the option for $19.58 and exercising it to get $20.00.

To adapt the Black-Scholes model to valuing American options, Fischer Black observed that the value of an American option is the greater of its Black-Scholes value or its early-exercise value. This method of valuing American options is now called Black's approximation.

Exercising today is one form of early exercise. In this example, according to Black's approximation, the value of this American option is $20.00.



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$82.44
$74.59
$67.49
$61.07
$55.26
$50.00
$45.24
$40.94
$37.04
$33.52
$30.33
$27.44
$24.83
$22.47
$20.33
$18.39
$16.64
$15.06
Days until ex-dividend
Dividend amount
First dividend 5 1.50
Second dividend 96 1.50
Third dividend
Fourth dividend
Fifth dividend
Sixth dividend
Seventh dividend
Eighth dividend
Calculate Net Present Value
Using Risk-free Rate
Using Expected Return
Dividend NPV 2.98 2.98
Close Dividend Schedule
To calculate dividend net present value, the model discounts the stream of dividends expected to be paid during the investment period.
The model calculates two different dividend net present values. For one, the model uses as the discount rate the risk-free rate of return. For the other, it uses the expected return.
Annualized
Period
Expected CC Return 6.00 3.0082%
Median CC Return -4.125% -2.0682%
Standard Deviation 45.00 31.8634%
Dividend NPV 2.98
CC Risk-free Rate 6.00 3.0082%
Current Asset Price 50.00
Days to Expire (365 D/Y) 183
Put Strike Price 70.00
Put Option Price
Put
Calculate Option Value
Fast
Fastest
Display Div Schedule
No Div

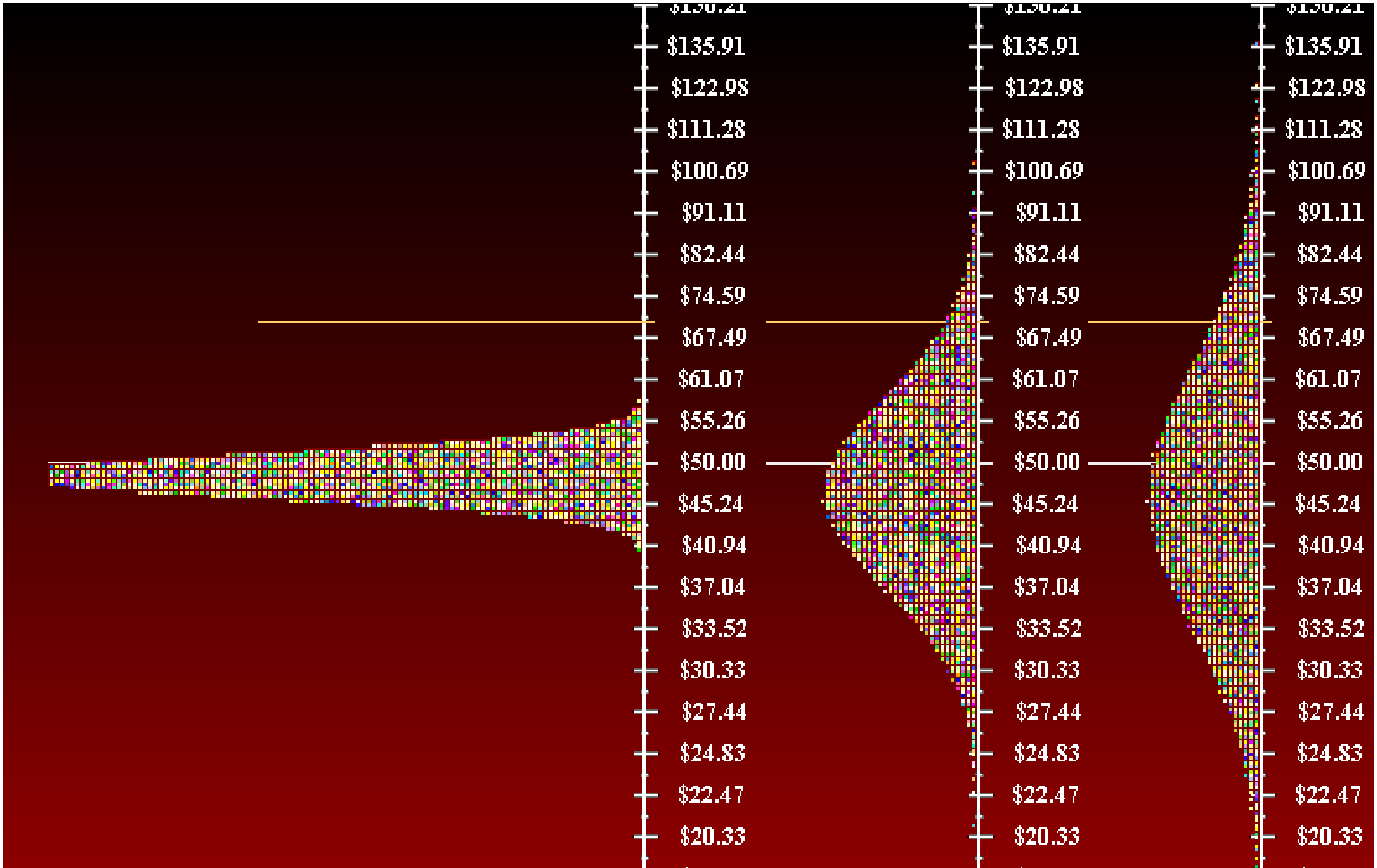
What if a put's underlying pays lumpy dividends? Under what circumstances is it logical to exercise the put early?

What if, in our example, we add dividends to the put's underlying? Dividend payments lower a stock's price which drives puts farther into the money.

Will it still be optimal to exercise the put today?

Let's see.

18. Click once on Clear.
19. Click on Display Div Schedule.
20. For First dividend, Days until ex-dividend, enter 5.
21. For First dividend, Dividend amount, enter 1.50.
22. For Second dividend, Days until ex-dividend, enter 96.
23. For Second dividend, Dividend amount, enter 1.50.
24. Click on Calculate Net Present Value.
25. Click on Close Dividend Schedule.
26. Click on Calculate Option Value Fast or Fastest.



Exercise	Today	In 5 days	In 96 days	In 183 days
Probability-weighted net present value	$20.00	$21.44	$22.15	$22.00

If a put goes deep in the money and the underlying pays dividends, it may be optimal to exercise the option on the last ex-dividend date

With two dividends of $1.50 being paid before expiration and with ex-dividend dates 5 and 96 days in the future, the probability-weighted net present value of holding the option until maturity is $22.00.

The exercise-today value is still $20.00. With the addition of dividends to the underlying, we no longer gain an advantage from exercising the option today.

Does holding the option until maturity offer the highest probability-weighted net present value?

Let's look at possible exercise on the two ex-dividend dates 5 and 96 days into the future.

27. For Days to Expire, enter 96.
28. Click on Calculate Option Value Fast or Fastest.

Exercise on the second ex-dividend date gives a probability-weighted net present value of $22.15—a higher value than for either exercising the option today or holding it until maturity.

Let's look at a possible exercise on the first dividend's ex-dividend date.

29. For Days to Expire, enter 5.
30. Click on Calculate Option Value Fast or Fastest.

Exercise on the first ex-dividend date gives a probability-weighted net present value of $21.44—more than for exercise today but less than for holding the option until maturity or for exercising it on the second ex-dividend date.

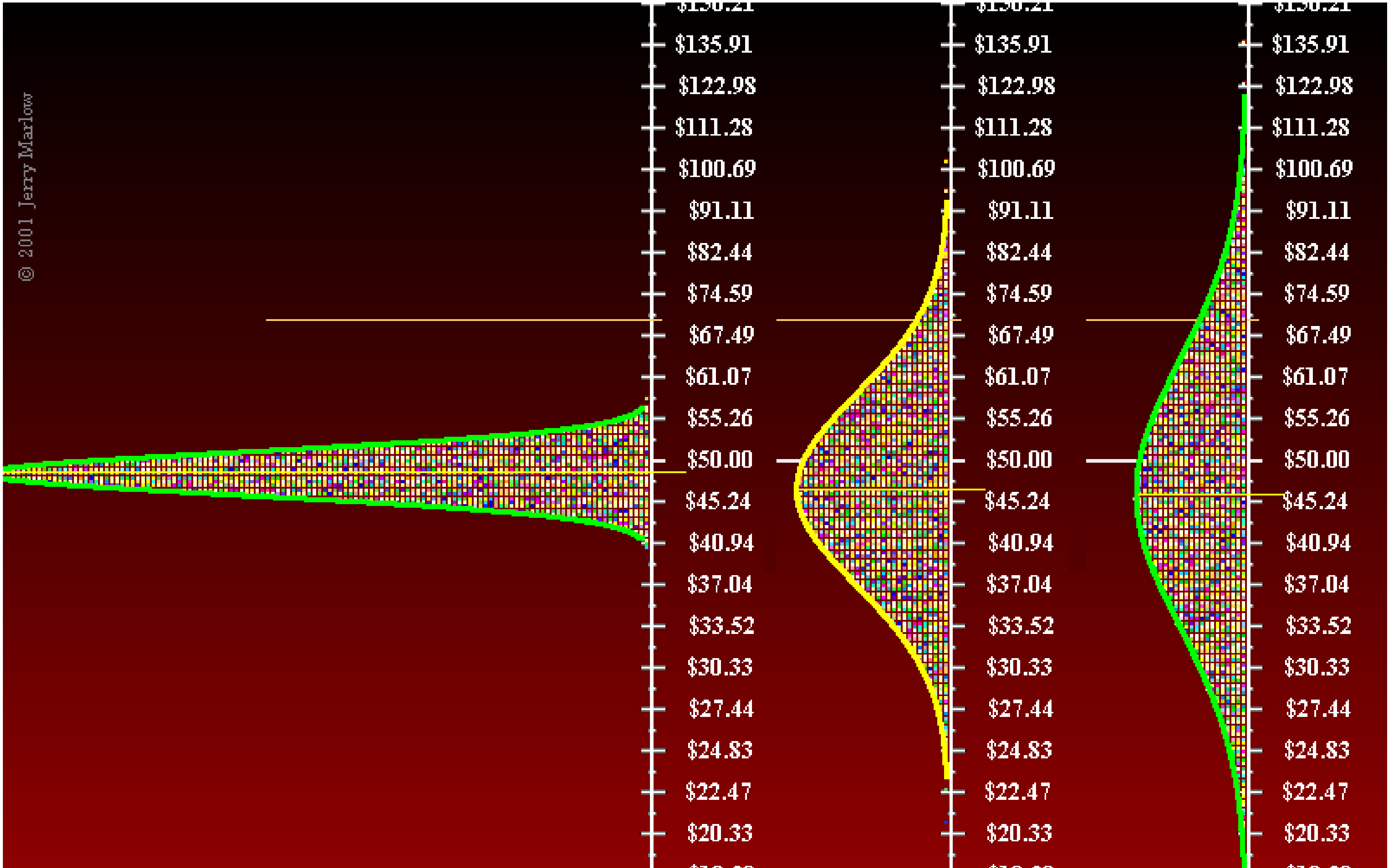
If a put is deep in the money and the underlying pays dividends, often it will be optimal to exercise the option on the underlying's last ex-dividend date.

What is the value of this option?

Since we've had the underlying's expected return set to the risk-free rate, the Black-Scholes value for each of the exercise dates is the same as the probability-weighted net present value.

Using Black's approximation, the option's value is the highest of the values for the four exercise dates—$22.15.

(To create the exhibit above and others like it, the author used Print Scrn to capture animation screens and pasted them into image-editing software. There he chopped up the captures and assembled them into exhibits. You can do the same.)



Exercise	Today	On first ex-div day	On last ex-div day	At maturity
Probability-weighted net present value	$20.00	$21.44	$22.15	$22.00

What's going on here? As the probability distribution shifts down relative to the strike price and changes shape, the payoff of each little square keeps changing.

If you stare long enough at the three probability distributions above, you will have an important insight: An option's probability-weighted net present value depends on a dynamic combination of factors. The probability-weighted net present value is determined both by how high or low the probability distribution is relative to the strike price and by how spread out it is.

You probably didn't notice, but when we set up this underlying, the annualized expected return of 6% and standard deviation of 45% gave us a median return of -4.125%. A negative median return means that, as we increase our investment horizon, the end-of-period probability distribution creeps down the price and return axes.

Little Squares on First Ex-div Date
In our example, when we look ahead just five days, the median return hasn't had enough time to sink the distribution very much.

We see a narrow probability distribution. There's not much uncertainty. The dividend payment has shifted the distribution down some.

The entire probability distribution is below the strike price. Every little square is in the money (as each would be even without the dividend payment).

The downward shift makes prospective payoffs at exercise in five days greater for all the little squares.

Looking just five days into the future, however, none of the squares are super deep into the money. There's almost no chance of the stock price going really low and generating a really big payoff.

Little Squares on Last Ex-div Date
When we look ninety-six days into the future at the second ex-dividend date, the negative median return has shifted the probability distribution down a tad. The dividend payment also has acted to shift it down. More of the little squares are below the strike price than there would be without the dividend payment.

We see substantially more uncertainty than we saw at five days. The distribution is much more spread out than at five days. Some of the little squares are out of the money. Others are very deep in the money.

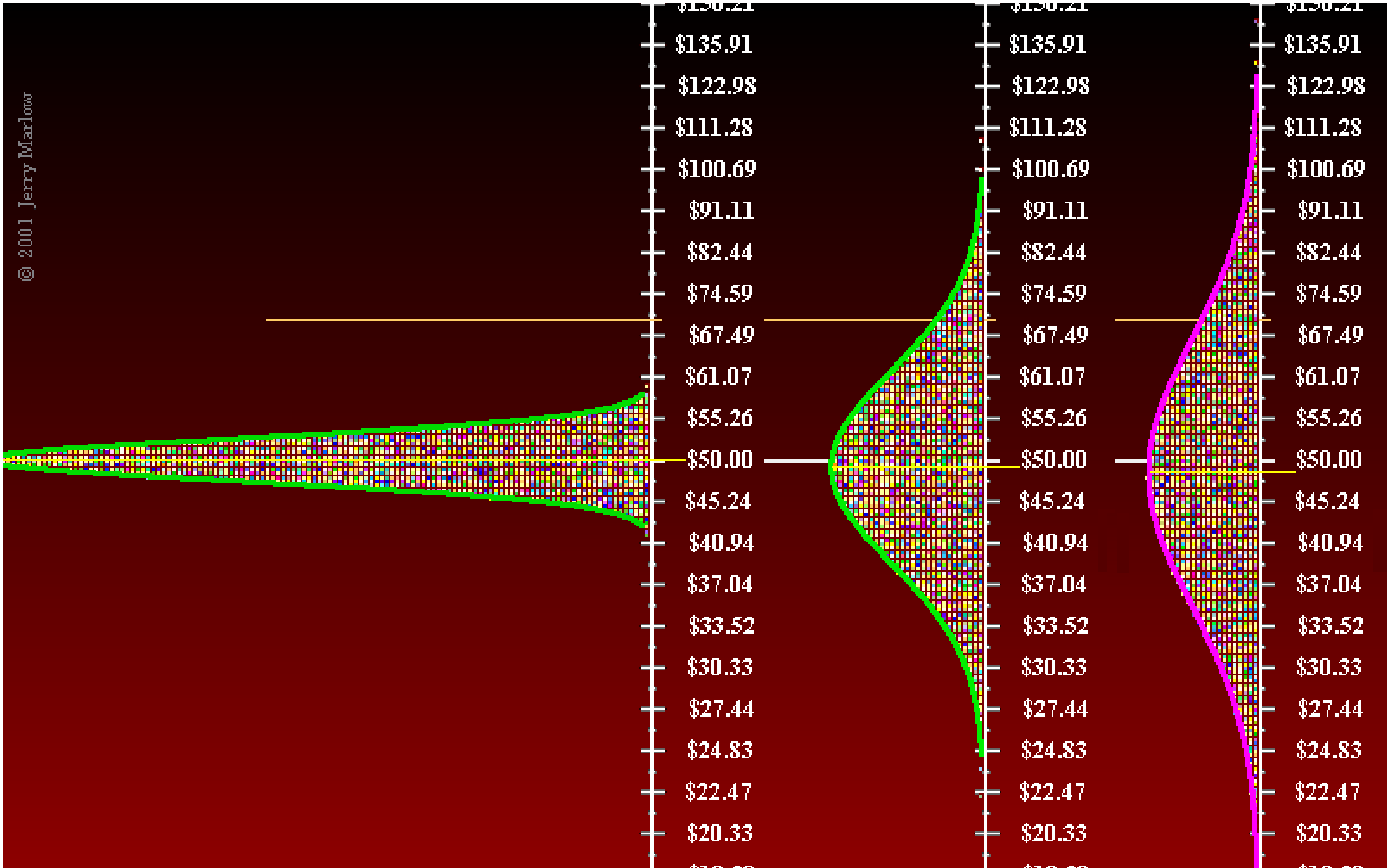
Little Squares at Maturity
When we look 183 days into the future, the negative median return has sunk the probability distribution slightly farther. There have been no more dividends.

We see more uncertainty. The distribution is more spread out. More of the little squares are out of the money. Some are deeper into the money.

Strike price leverages value of each little square
For all three horizons, we're looking at probability distributions for the underlying. Option structures leverage the outcome for every little square. Options magnify the effect of lifting, falling, and changes in shape of the underlying's probability distribution.

How much an option leverages the outcome of each little square depends on where that little square is in relation to the strike price.

As you peer different distances into the future, the option's probability-weighted net present value tells you the net effect on all the little squares of the rises, falls, and changes in shape.



Exercise	Today	In 5 days	In 96 days	In 183 days
PWNPV				
With 2% Div Yield	$20.00	$19.96	$19.65	$19.97
With 3% Div Yield	$20.00	$19.96	$19.76	$20.17

If a put goes deep in the money and the underlying pays a dividend yield, the effect on the option's probability-weighted net present value is complex. You're on your own!

Instead of paying lumpy dividends, let's say an underlying pays a dividend yield. What are the early-exercise implications for deep-in-the-money puts?

In our example, we'll keep everything else the same and look at two different yields: 2% and 3%.

31. Click on Clear.
32. Click on Enter Div Yield.
33. For Continuous Div Yield, enter 3.00.
34. For Days to Expire, enter 5.
35. Click on Calculate Option Value Fastest.

Note the option's probability-weighted net present value.

36. For Continuous Div Yield, enter 2.00.
37. Click on Draw Your Forecast.
38. Click on Calculate Black-Scholes Value.

(Because the expected return equals the risk-free rate, calculating the Black-Scholes value is the same as calculating the probability-weighted net present value.)

The probability distributions are so similar as to be indistinguishable. At five days, we get the same probability-weighted net present value for both dividend yields: $19.96.

So far, you're better off if you exercise the put today.

We do the same thing for 96 days.

39. Click on Clear.
40. For Continuous Div Yield, enter 3.00.
41. For Days to Expire, enter 96.
42. Click on Calculate Option Value Fastest.

Note the option's value.

43. For Continuous Div Yield, enter 2.00.
44. Click on Draw Your Forecast.
45. Click on Calculate Black-Scholes Value.

At ninety-six days, the probability-weighted net present values are $19.65 for the 2% yield and $19.76 for the 3%.

For both puts, you're better off if you exercise today.

We do the same thing for 183 days which we've taken to be the put's expiration date.

46. Click on Clear.
47. For Continuous Div Yield, enter 3.00.
48. For Days to Expire, enter 183.
49. Click on Calculate Option Value Fastest.

Note the option's value.

50. For Continuous Div Yield, enter 2.00.
51. Click on Draw Your Forecast.
52. Click on Calculate Black-Scholes Value.

At 183 days, the probability-weighted net present value for the 2% yield is $19.97. Exercising today would still give you a higher present value.

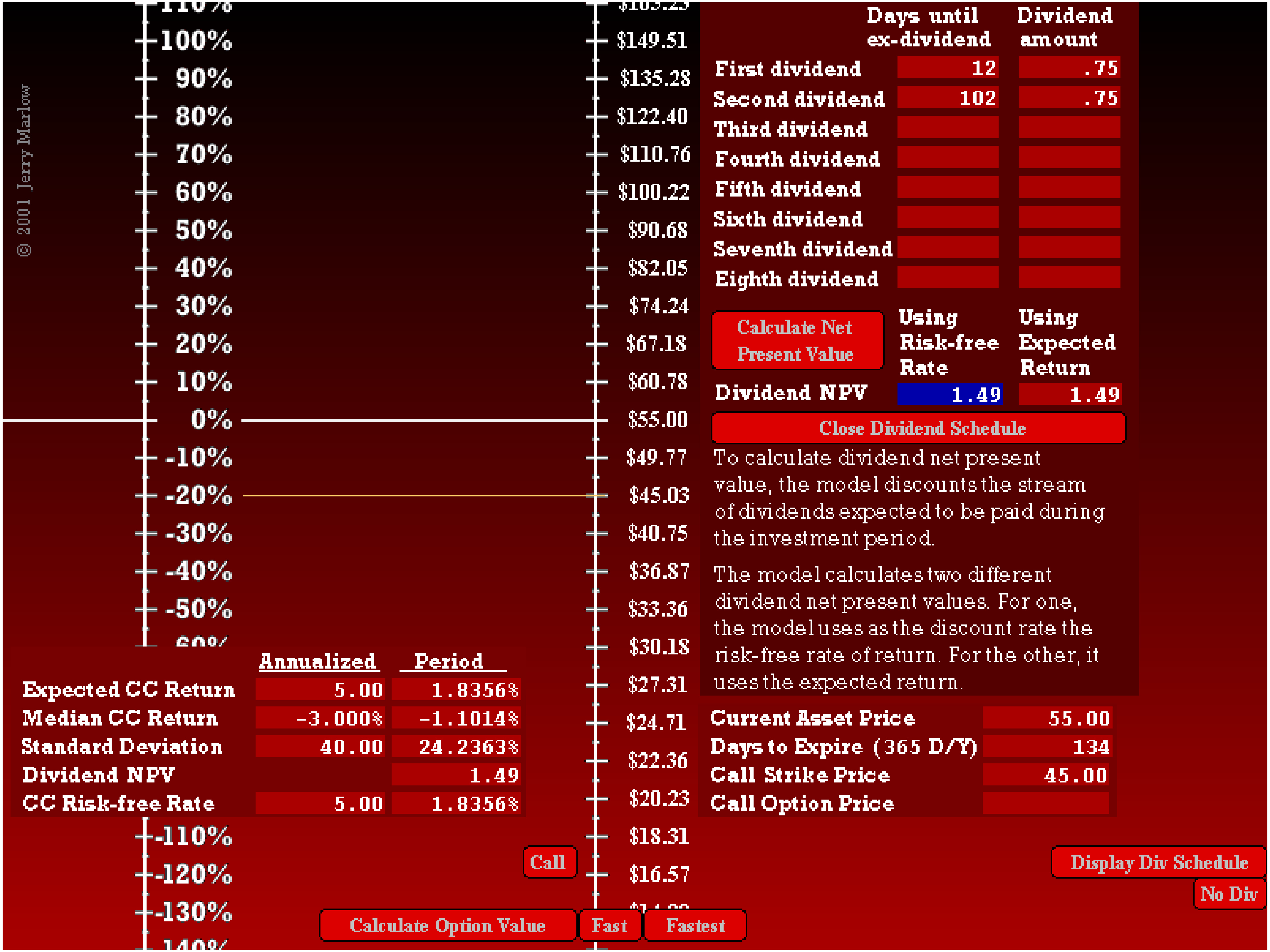
For the 3% yield, however, the probability-weighted net present value has gone to $20.17—more than the exercise-today payoff.

The probability distributions diverge in subtle ways. The values of both puts have gone down and then back up. After 183 days, the values of both puts would continue to grow. Were you to hold the put for the 2% yield seven more days, its value would go to $20.00.

What's the right thing to do under these circumstances?

Most authors duck the issue.

Me too.



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$67.18
$60.78
$55.00
$49.77
$45.03
$40.75
$36.87
$33.36
$30.18
$27.31
$24.71
$22.36
$20.23
$18.31
$16.57
Days until ex-dividend
Dividend amount
First dividend 12 .75
Second dividend 102 .75
Third dividend
Fourth dividend
Fifth dividend
Sixth dividend
Seventh dividend
Eighth dividend
Calculate Net Present Value
Using Risk-free Rate
Using Expected Return
Dividend NPV 1.49 1.49
Close Dividend Schedule
To calculate dividend net present value, the model discounts the stream of dividends expected to be paid during the investment period.
The model calculates two different dividend net present values. For one, the model uses as the discount rate the risk-free rate of return. For the other, it uses the expected return.
Annualized
Period
Expected CC Return 5.00 1.8356%
Median CC Return -3.000% -1.1014%
Standard Deviation 40.00 24.2363%
Dividend NPV 1.49
CC Risk-free Rate 5.00 1.8356%
Current Asset Price 55.00
Days to Expire (365 D/Y) 134
Call Strike Price 45.00
Call Option Price
Call
Display Div Schedule
No Div
Calculate Option Value
Fast
Fastest

If its underlying pays lumpy dividends and a call goes deep into the money, when should you exercise it?

In the case of a deep-in-the-money put on an underlying that pays dividends, we saw that the put value increased whenever a dividend payment pushed the underlying's probability distribution down. In our example, we saw that the best time to exercise the put was immediately *after* the dividend.

For a call option, when an underlying's dividend payments push the probability distribution down, it pushes more little squares out of the money. When might the time of optimal exercise be?

Let's see.

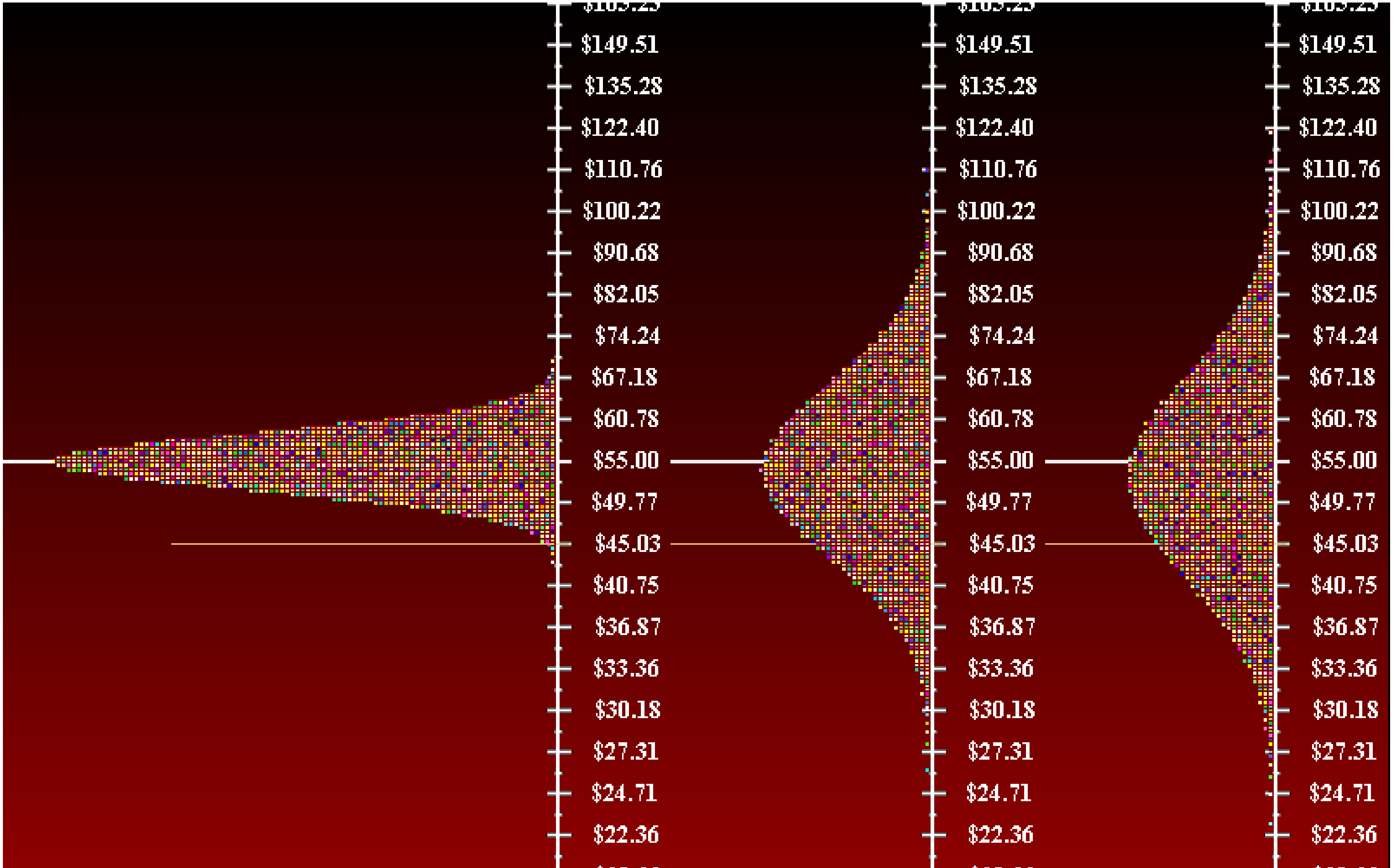
1. Click three times on Clear.
2. Click twice on Hide.
3. Click on Call.
4. Click on Calculate Option Value.

To simplify things a little, we're going to make the underlying expected return and the risk-free rate the same.

5. For Expected CC Return, enter 5.00.
6. For Standard Deviation, enter 40.00.

This combination of expected return and standard deviation gives us a negative median return. Over time, the negative return will push the middle of the underlying's probability distribution down.

7. For Current Asset Price, enter 55.00.
8. For Days to Expire, enter 134.
9. For Call Strike Price, enter 45.00.
10. Click on Display Div Schedule.
11. For CC Risk-free Rate, enter 5.00.
12. For First dividend, Days until ex-dividend, enter 12.
13. For First dividend, Dividend amount, enter 0.75.
14. For Second dividend, Days until ex-dividend, enter 102.
15. For Second dividend, Dividend amount, enter 0.75.
16. Click on Calculate Net Present Value.
17. Click on Close Dividend Schedule.
18. Click on Calculate Option Value Fastest.



Exercise	Today	Day before first div	Day before last div	At maturity
Probability-weighted net present value	$10.00	$10.07	$10.80	$10.76

If a call goes deep into the money and the underlying pays lumpy dividends, it may be optimal to exercise the option on the last day before the underlying goes ex-dividend for the last time

The probability-weighted net present value of the option's possible payoffs at maturity is $10.76.

If you were to exercise the option today, your payoff from early exercise would be $10.00. The option has time value. On average, it promises to be worth more when held to maturity than when exercised today.

Let's look at the probability-weighted net present value on the day before the first ex-dividend date, day 11.

19. Tab to the Days to Expire field. Enter 11.
20. Click on Calculate Option Value Fastest.

The probability-weighted net present value is $10.07—more than the exercise-today value, less than the value for exercise at maturity.

The underlying's last ex-dividend date is 102 days away. On that day, the dividend will push the underlying's probability distribution down. Let's look at the probability-weighted net present value of the underlying's probability distribution on the day before, on day 101.

21. For Days to Expire, enter 101.
22. Click on Calculate Option Value Fastest.

The probability-weighted net present value is $10.80—four cents more than for holding the option until maturity.

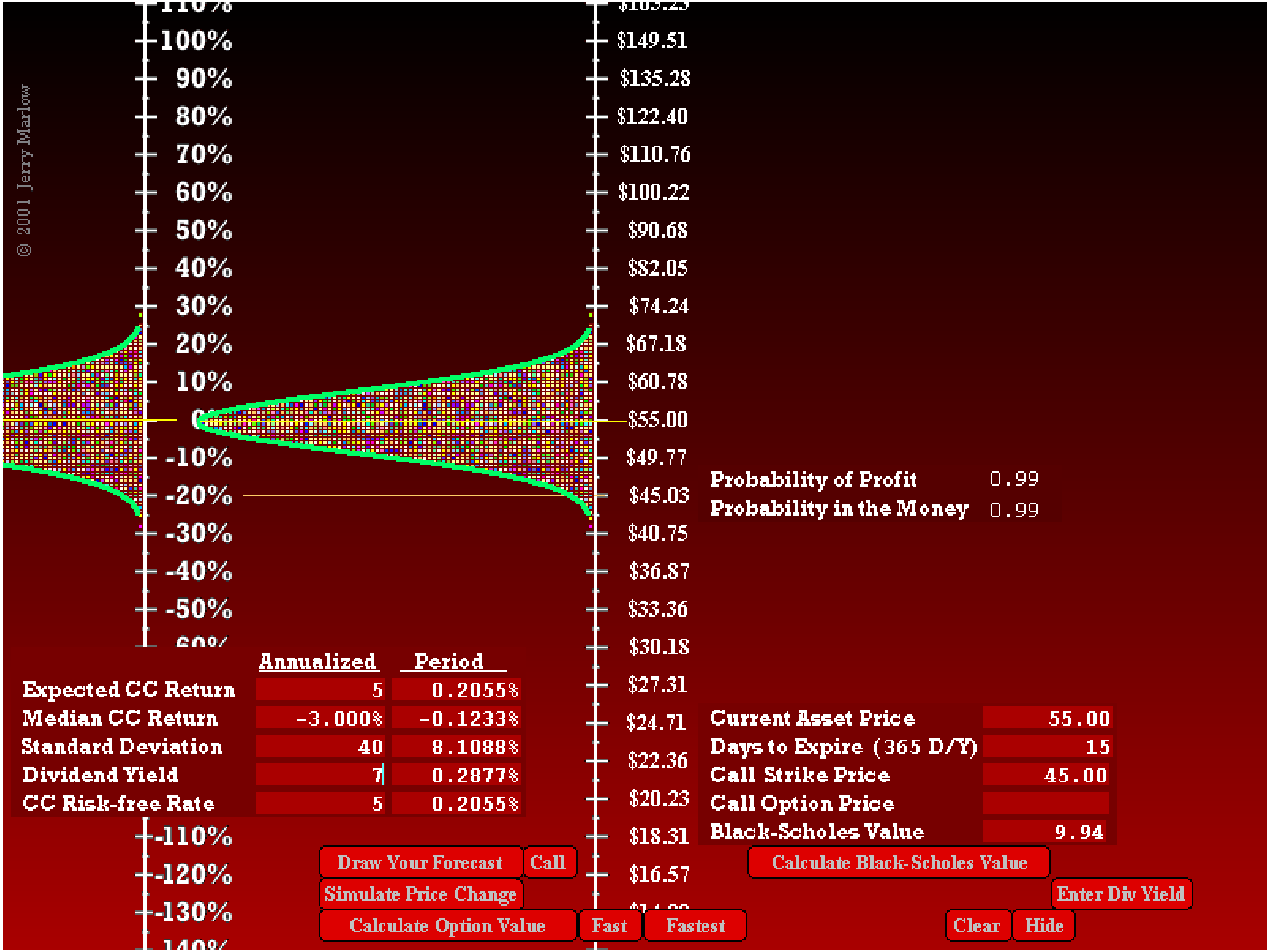
Looking into the future, right now it looks like the optimal time to exercise this option is on the last day before the underlying goes ex-dividend for the last time.

Using Black's approximation, we would value this option at $10.80—the highest Black-Scholes value of its possible exercise times.

Often when a call goes deep into the money and the underlying pays lumpy dividends, it is optimal to exercise the option on the last day before the underlying goes ex-dividend for the last time.

What we've seen is a common pattern, but not a rule on which you can depend. You have to do the analysis.

The outcomes vary with your forecast for the underlying's expected return—or with the risk-free rate if you use it as the expected return. Outcomes vary with your forecast for the underlying's volatility and with the size of the predicted dividend payments.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$74.24
$67.18
$60.78
$55.00
$49.77
$45.03
$40.75
$36.87
$33.36
$30.18
$27.31
$24.71
$22.36
$20.23
$18.31
$16.57
Probability of Profit 0.99
Probability in the Money 0.99
Annualized
Period
Expected CC Return 5 0.2055%
Median CC Return -3.000% -0.1233%
Standard Deviation 40 8.1088%
Dividend Yield 7 0.2877%
CC Risk-free Rate 5 0.2055%
Current Asset Price 55.00
Days to Expire (365 D/Y) 15
Call Strike Price 45.00
Call Option Price
Black-Scholes Value 9.94
-110%
-120%
-130%
Draw Your Forecast
Call
Simulate Price Change
Calculate Option Value
Fast
Fastest
Calculate Black-Scholes Value
Enter Div Yield
Clear
Hide

If the underlying pays a dividend yield and a call goes deep into the money, what determines the best time to exercise the option?

Let's look at how a dividend yield might affect a call's probability-weighted net present values for different times to exercise. As examples, we'll look at dividend yields of 5% and 7% for times to exercise of 0, 15, 30, 60, and 120 days.

Except for the times to exercise and a dividend yield instead of lumpy dividends, we keep the setting from the previous example. The spot price of $55 and strike price of $45 give us an exercise-today payoff of $10.

23. Click once on Clear.
24. Click on Enter Div Yield.
25. For Days to Expire, enter 15.
26. For Continuous Div Yield, enter 5.00.
27. Click on Calculate Option Value Fastest.

Note the probability-weighted net present value.

28. For Continuous Div Yield, enter 7.00.
29. Click on Draw Your Forecast.
30. Click on Calculate Black-Scholes Value.

The probability distributions are so similar as to be indistinguishable.

Looking ahead to exercise in fifteen days, a dividend yield of 5% gives us a value of $9.99. A dividend yield of 7% gives us a value of $9.94. Both values are below the exercise-today value of $10.00.

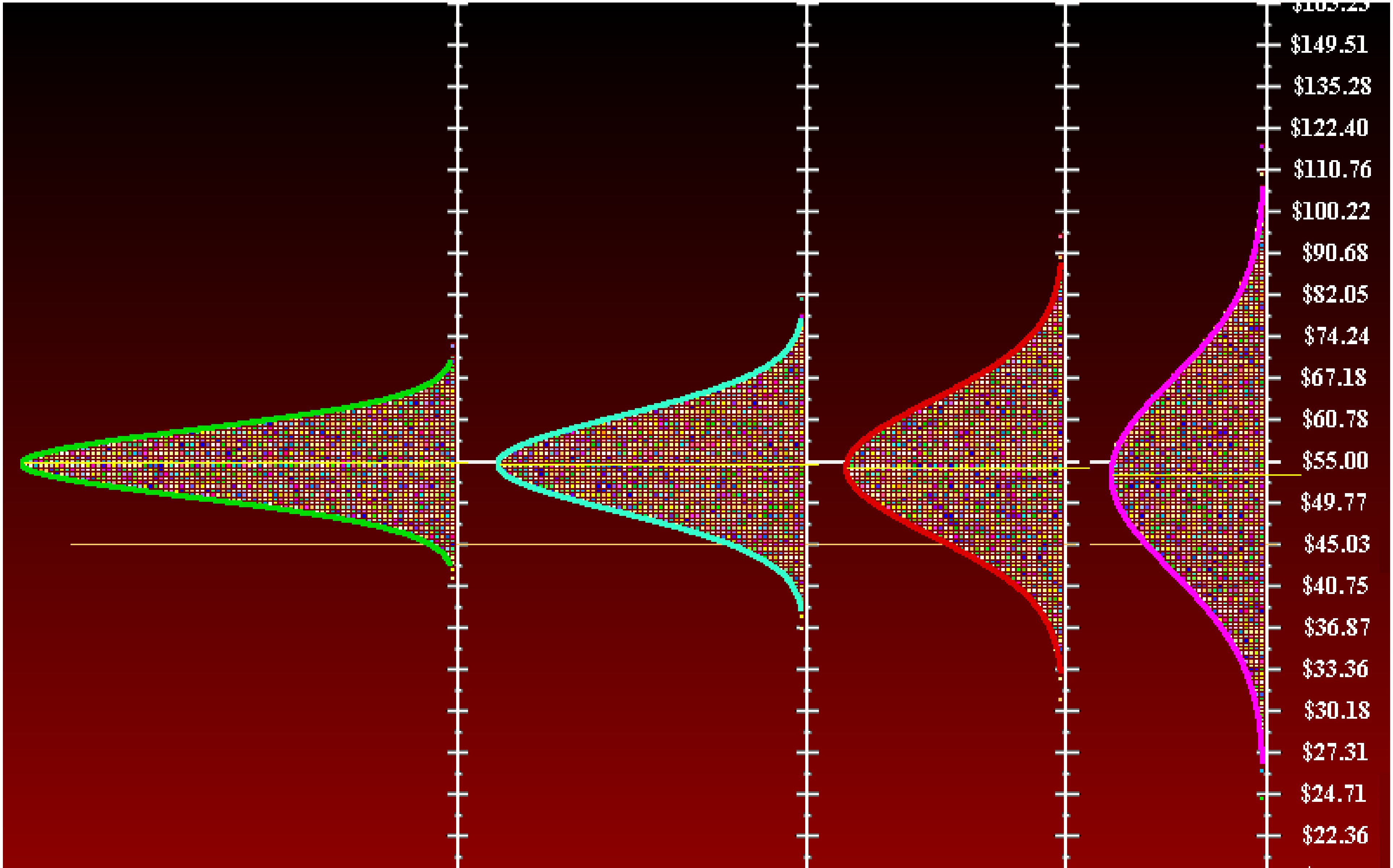
31. Click once on Clear.
32. For Days to Expire, enter 30.
33. For Continuous Div Yield, enter 5.00.
34. Click on Calculate Option Value Fastest.

Note the value.

35. For Continuous Div Yield, enter 7.00.
36. Click on Draw Your Forecast.
37. Click on Calculate Black-Scholes Value.

At 30 days, a 5% dividend yield gives us a value of $10.05. We're above the exercise-today value.

A 7% yield gives us a value of $9.96.



Exercise	Today	In 15 days	In 30 days	In 60 days	In 120 days
PWNPV					
5% Div Yield	$10.00	$9.99	$10.05	$10.33	$11.01
7% Div Yield	$10.00	$9.94	$ 9.96	$10.17	$10.72

If a call goes deep into the money and the underlying pays a dividend yield, the time of optimal exercise depends on the size of the yield, the option's remaining time value, the underlying's volatility, and its expected return or the risk-free rate

38. Click once on Clear.
39. For Days to Expire, enter 60.
40. For Continuous Div Yield, enter 5.00.
41. Click on Calculate Option Value Fastest.

Note the value.

42. For Continuous Div Yield, enter 7.00.
43. Click on Draw Your Forecast.
44. Click on Calculate Black-Scholes Value.

At 60 days, a 5% dividend yield gives us a value of $10.33. A 7% yield give us a value of $10.17. For both yields, we're above the exercise-today value.

45. Click once on Clear.
46. For Days to Expire, enter 120.
47. For Continuous Div Yield, enter 5.00.
48. Click on Calculate Option Value Fastest.

Note the value.

49. For Continuous Div Yield, enter 7.00.
50. Click on Draw Your Forecast.
51. Click on Calculate Black-Scholes Value.

At 120 days, a 5% dividend yield gives us a value of $11.01. A 7% yield give us a value of $10.72.

In the progression from exercise today to exercise in 120 days, the probability distributions for 5% and 7% are visually indistinguishable.

Over time, the negative median returns and dividend yields push the underlying's probability distributions down the price axis.

From the calculated values, we see that the higher the dividend yield is, the lower the value of the call.

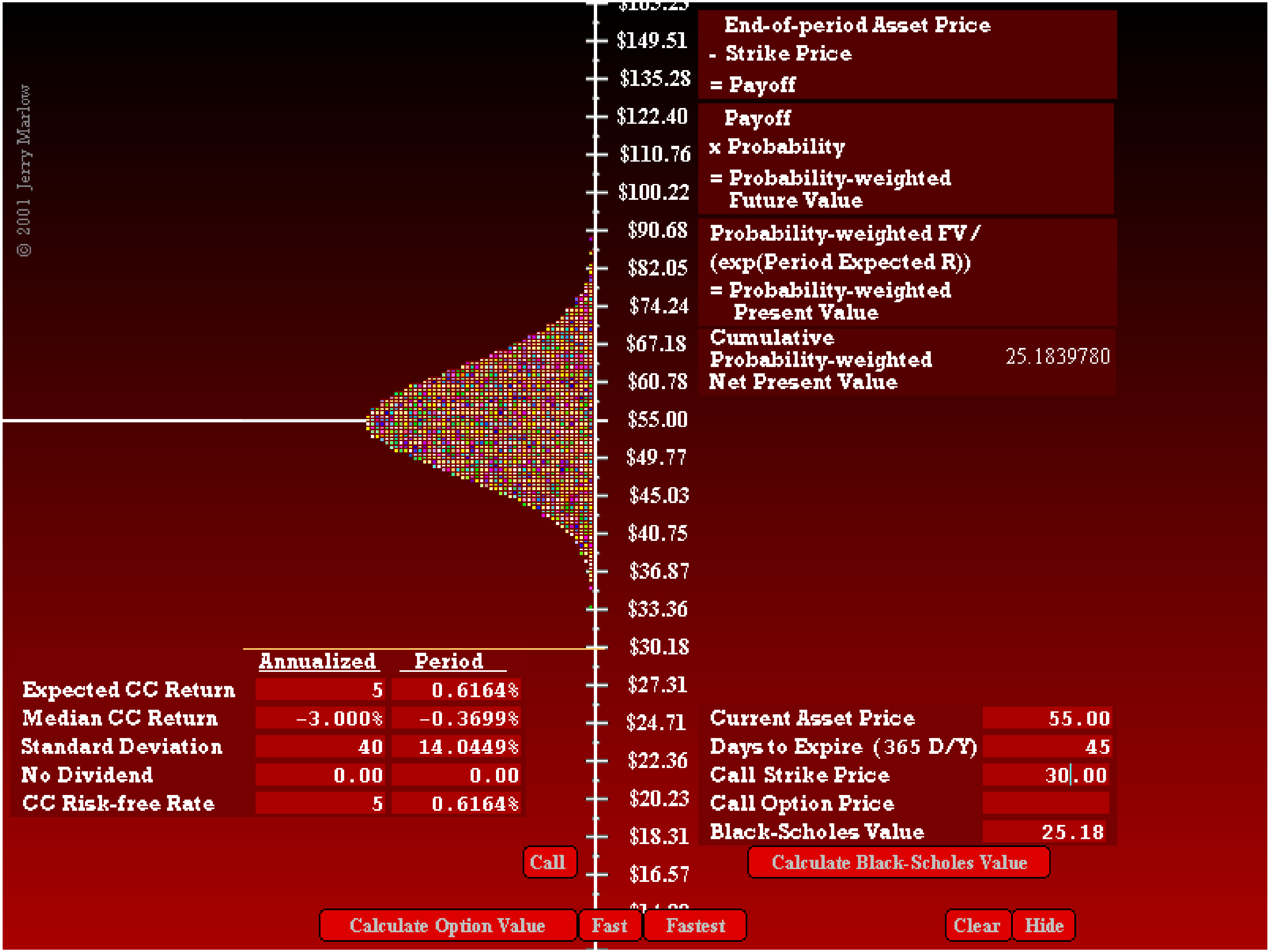
Early on, the tails of the probability distributions below the strike price drag the averages of the little squares' payoffs below the exercise-today values.

Later, the higher and higher payoffs from the outcomes at the tops of the probability distributions more than compensate for the absence of payoffs from little squares below the strike price.

With 120 days to expiration, if you don't mind the intervening dip in value, it makes sense to hold on to both options. Had the options only 15 days until expiration, it would be logical to exercise both of them today.

The examples show the complexity of finding the right time to exercise calls on underlyings that pay dividend yields.

Using Black's approximation and times to expiration of 120 days, the value of the call on the underlying with the 5% dividend yield is $11.01. For the 7% dividend yield, it is $10.72.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$45.03
$40.75
$36.87
$33.36
$30.18
$27.31
$24.71
$22.36
$20.23
$18.31
$16.57
End-of-period Asset Price
- Strike Price
= Payoff
Payoff
x Probability
= Probability-weighted Future Value
Probability-weighted FV / (exp(Period Expected R))
= Probability-weighted Present Value
Cumulative Probability-weighted Net Present Value
25.1839780
Annualized
Period
Expected CC Return
5
0.6164%
Median CC Return
-3.000%
-0.3699%
Standard Deviation
40
14.0449%
No Dividend
0.00
0.00
CC Risk-free Rate
5
0.6164%
Current Asset Price
55.00
Days to Expire (365 D/Y)
45
Call Strike Price
30.00
Call Option Price
Black-Scholes Value
25.18
Call
Calculate Black-Scholes Value
Calculate Option Value
Fast
Fastest
Clear
Hide

If you own a call on an underlying that pays no dividends, when does exercising it early give you an advantage? Never!

Now let's do something that's a lot easier—look at a call on an underlying that pays no dividends. The call is super deep into the money.

52. Click once on Clear.
53. Click on No Div.
54. For Days to Expire, enter 45.
55. For Call Strike Price, enter 30.00.
56. Click on Calculate Option Value Fastest.
57. Click on Calculate Black-Scholes Value.

You cannot get much deeper into the money. The entire probability distribution is above the strike price.

Still the probability-weighted net present value is eighteen cents higher than the exercise-today value.

Under ordinary conditions, on an underlying that pays no dividends, you can never have a call option that has no time value. To see for yourself, try different numbers of days to expiration, different strike prices, and different risk-free rates.

If you're calculating the option's Black-Scholes value, the only way to get the time value to be zero or negative is to set the risk-free rate to zero or below.

If you're using expected return to calculate the option's probability-weighted net present value, if you set the expected return below zero, the time value will go negative. If you set the strike price equal to zero, the option value will equal the exercise-today value, which will be equal to the spot price of the underlying.

On underlyings that pay no dividends, European calls and American calls have the same value. You can use Black-Scholes to value them without using Black's approximation.

Some American options have more value than otherwise identical European options because of the potential value of early exercise. Since the right to early exercise has no value for a call written on an underlying that pays no dividends, the right of early exercise has no value.

On these underlyings, European and American calls have the same value. Instead of looking at possible early-exercise points and using Black's approximation, you can use Black-Scholes to value them. You can do other types of analyses that rely on the Black-Scholes assumptions.

Sensitivity of option values to changes in volatility, spot price of underlying, time to expiration, and risk-free rate

Factors that change an option's value change how many little squares are above or below the strike price and how far they are above it or below it

The value of an option is determined by where all those little squares in the probability distribution of the underlying fall relative to the option's strike price. Factors that change where they fall change the option's value.

Changes in the spot price of the underlying, the option's time to expiration, the underlying's expected volatility, and the risk-free rate all change the underlying's probability distribution. When these factors change, the probability distribution of the underlying may go up or down, spread out or contract. The number of little squares above or below the strike price increases or decreases accordingly. The distance of the little squares from the strike price—and, hence, the size of their payoffs—may increase or decrease. The value of the option changes accordingly.

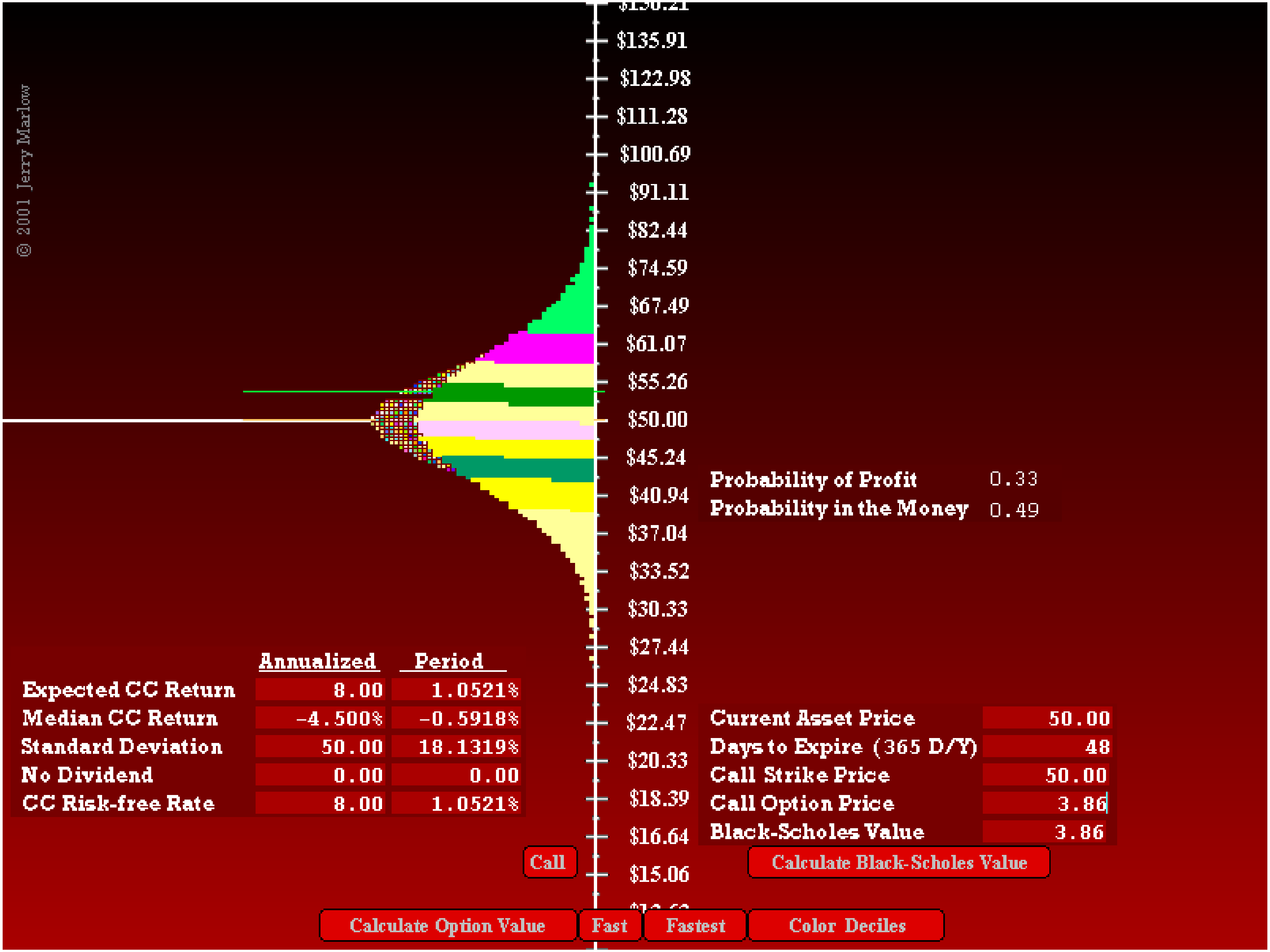
Many books and tutorials on options quickly launch into a discussion of the Greeks—the symbols for how much an option's value changes with changes in the spot price of the underlying, the option's time to expiration, the underlying's volatility, and the risk-free rate. They give you a calculator or formula for calculating the Greeks and act as if they've given you something of great value.

We're not going to do that.

Once you're an experienced options trader, you'll understand the Greeks and they will be of value to you. You'll use them to set up and manage hedges for options you sell. You'll use them to help manage your risk exposures.

If you're just learning about options, a more useful approach, which we will follow here, is to look at how changes in different factors change where the little squares fall relative to the option's strike price.

Calculation of the Greeks under the Black-Scholes assumptions assumes that all options are European-style. We'll follow that assumption.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$37.04
$33.52
$30.33
$27.44
$24.83
$22.47
$20.33
$18.39
$16.64
$15.06
Probability of Profit 0.33
Probability in the Money 0.49
Annualized Period
Expected CC Return 8.00 1.0521%
Median CC Return -4.500% -0.5918%
Standard Deviation 50.00 18.1319%
No Dividend 0.00 0.00
CC Risk-free Rate 8.00 1.0521%
Current Asset Price 50.00
Days to Expire (365 D/Y) 48
Call Strike Price 50.00
Call Option Price 3.86
Black-Scholes Value 3.86
Call
Calculate Black-Scholes Value
Calculate Option Value
Fast
Fastest
Color Deciles

The deeper an option goes into the money, the less sensitive it is to changes in the factors that determine its value

How much a change in spot price, time to expiration, volatility, or the risk-free rate changes an option's value depends on how much it alters the position of the little squares in the underlying's probability distribution relative to the strike price.

If an option is deep in the money or at-the-money, a change in one of the factors does not change very much the proportion of squares above and below the strike price. The percentage change in option value is relatively small.

If, for instance, the volatility of an at-the-money call option increases by 25%, the proportion of squares above and below the strike price changes very little.

1. Click three times on Clear.
2. Click twice on Hide.
3. Click on Call.
4. Click on Calculate Option Value Fast.
5. Click on Calculate Black-Scholes Value.
6. For Expected CC Return, enter 8.00.
7. For Standard Deviation, enter 40.00.
8. For CC Risk-free Rate, enter 8.00.
9. For Current Asset Price, enter 50.00.
10. For Days to Expire, enter 48.
11. For Call Strike Price, enter 50.00.

This is an at-the-money option. The strike price is equal to the spot price.

We've set it up so that the expected return equals the risk-free rate and the median return is zero.

12. Click on Calculate Black-Scholes Value.

The option's Black-Scholes value is $3.14.

13. For Call Option Price, enter $3.14.
14. Click on Color Deciles.

The probability of being in the money is 0.50. That means fifty percent of the little squares are above the strike price; fifty percent, below.

Let's see how much the proportion of little squares above the strike price changes when we increase the standard deviation by twenty-five percent.

15. Click on Calculate Option Value Fastest.
16. For Standard Deviation, enter 50.00.

(Increasing a standard deviation of 40.00 by 10.00 is a 25% change:

10/40=25%.)

17. Click on Calculate Black-Scholes Value.

The option's Black-Scholes value is $3.86.

18. For Call Option Price, enter $3.86.
19. Click on Color Deciles.

We've gone from a 0.50 probability of being in the money to.49. That's a change of

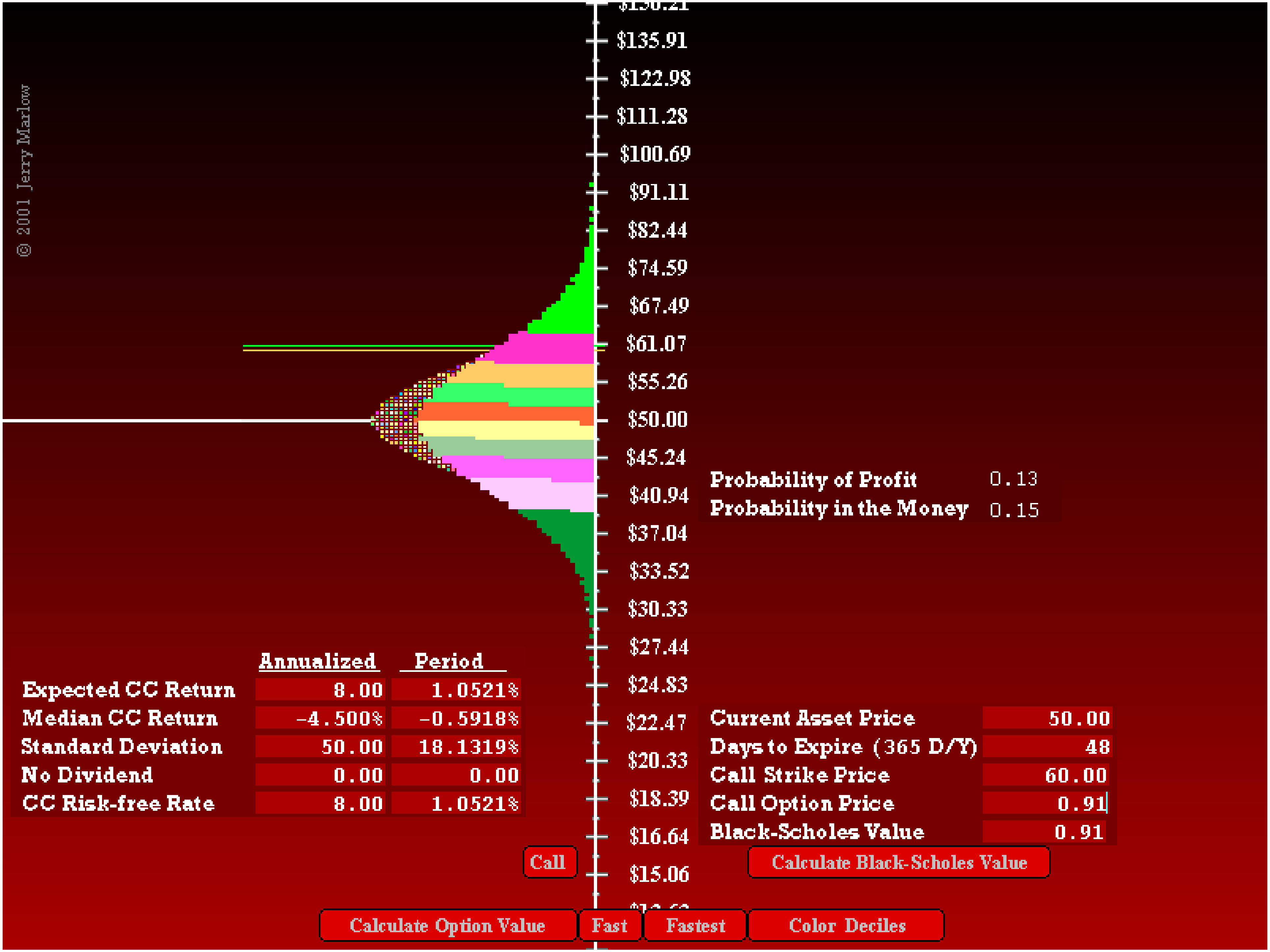
(.49 - 0.50)/.50 = -2%

Let's see the percentage change in the option's value :

($3.86 - $3.14)/$3.14 = 23%

Thus, in this instance, a 25% change in expected volatility gives us a 23% increase in option value.

Let's compare these sensitivities to those of an option that is otherwise identical but that is far out of the money.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$22.47
$20.33
$18.39
$16.64
$15.06
Probability of Profit 0.13
Probability in the Money 0.15
Annualized Period
Expected CC Return 8.00 1.0521%
Median CC Return -4.500% -0.5918%
Standard Deviation 50.00 18.1319%
No Dividend 0.00 0.00
CC Risk-free Rate 8.00 1.0521%
Current Asset Price 50.00
Days to Expire (365 D/Y) 48
Call Strike Price 60.00
Call Option Price 0.91
Black-Scholes Value 0.91
Call
Calculate Black-Scholes Value
Calculate Option Value
Fast
Fastest
Color Deciles

The farther an option is out of the money, the more sensitive it is to changes in the factors that affect its value

Now we look at the sensitivities of an option that is otherwise the same but far out of the money. Instead of starting with fifty percent of the little squares above the strike price, we start with ten percent above.

20. Click on Clear.
21. For Standard Deviation, enter 40.00.
22. For Call Strike Price, enter 60.00.
23. Click on Calculate Black-Scholes Value.

The option's Black-Scholes value is $0.46.

24. For Call Option Price, enter 0.46.
25. Click on Color Deciles.

The probability of being in the money is .10. That means ten percent of the little squares are above the strike price; ninety percent, below.

Let's see how much the proportion of little squares above the strike price changes this time when we increase the standard deviation by twenty-five percent.

26. Click on Calculate Option Value Fastest.
27. For Standard Deviation, enter 50.00.
28. Click on Calculate Black-Scholes Value.

The option's Black-Scholes value is $0.91.

29. For Call Option Price, enter 0.91.
30. Click on Color Deciles.

We've gone from a .10 probability of being in the money to.15. That's a change of
(0.15 - .10)/.10 = 50%

Let's see the percentage change in the option's value :

($0.91 - $0.46)/$0.46 = 98%

Thus we see, changes in the factors that affect option value have their greatest effects on options that are far out of the money.

ς, vega—If volatility increases, more little squares are above the strike price. Little squares are farther above the strike price. The value of a call option goes up

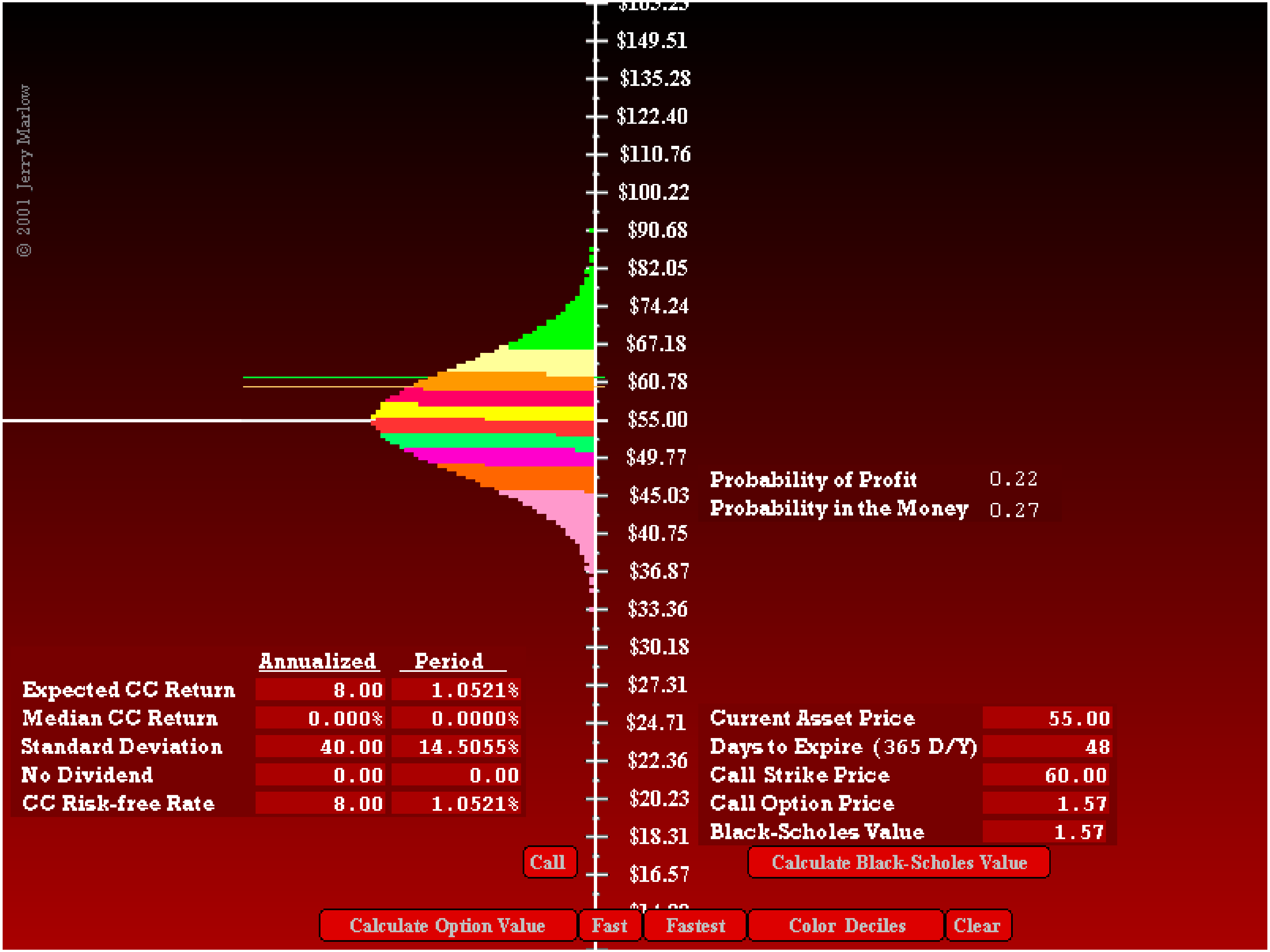
One of the Black-Scholes assumptions is that expected volatility of the underlying is constant over the life of an option. However, the expected volatility may change.

ς, vega, is the measure of how sensitive an option's price is to changes in the volatility of the underlying.

In our examples above, when we increased the volatility of the underlying, the probability distribution spread out. More of the little squares went above the strike price. Those already above went farther above. The value of the options went up.

What we saw holds as a general rule: When all else stays the same, an increase in volatility increases the value of a call option.

Keep in mind that an increase in volatility lowers the underlying's median return. In our examples, the increase changed the median return from zero to -4.50%.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Probability of Profit 0.22
Probability in the Money 0.27
Annualized Period
Expected CC Return 8.00 1.0521%
Median CC Return 0.000% 0.0000%
Standard Deviation 40.00 14.5055%
No Dividend 0.00 0.00
CC Risk-free Rate 8.00 1.0521%
Current Asset Price 55.00
Days to Expire (365 D/Y) 48
Call Strike Price 60.00
Call Option Price 1.57
Black-Scholes Value 1.57
Call
Calculate Black-Scholes Value
Calculate Option Value
Fast
Fastest
Color Deciles
Clear

Δ, *delta—When the spot price of the underlying increases, more little squares are above the strike price. Little squares are farther above it. The value of a call goes up.*

Delta is the rate of change of the option price with respect to the price of the underlying asset. Let's see how a change in the underlying's spot price changes where the little squares fall relative to a call option's strike price.

First we redraw our baseline call option which has a value of $0.46.

1. Click twice on Clear.
2. Click twice on Hide.
3. Click on Call.
4. Click on Calculate Option Value Fast.
5. Click on Calculate Black-Scholes Value.
6. Tab to Expected CC Return. Enter 8.00.
7. Tab to Standard Deviation. Enter 40.00.
8. For CC Risk-free Rate, enter 8.0.
9. For Current Asset Price, enter 50.00.
10. For Days to Expire, enter 48.
11. For Call Strike Price, enter 60.00.
12. Click on Calculate Black-Scholes Value.

The option's Black-Scholes value is $0.46.

13. For Call Option Price, enter 0.46.
14. Click on Color Deciles.

Ten percent of the little squares are above the strike price.

Let's see how a change in the underlying's spot price changes the value of the call. We'll increase the asset price by ten percent.

15. For Current Asset Price, enter 55.00.
16. Click on Calculate Black-Scholes Value.

The option value is $1.57.

17. For Call Option Price, enter 1.57.
18. Click on Color Deciles.

The shape of the probability distribution remains the same.

Now twenty-seven percent of the little squares are above the strike price. That's an increase of

(.27 - .10)/.10 = 170%

The option's value increased by

($1.57 - $0.46)/$0.46 = 241%

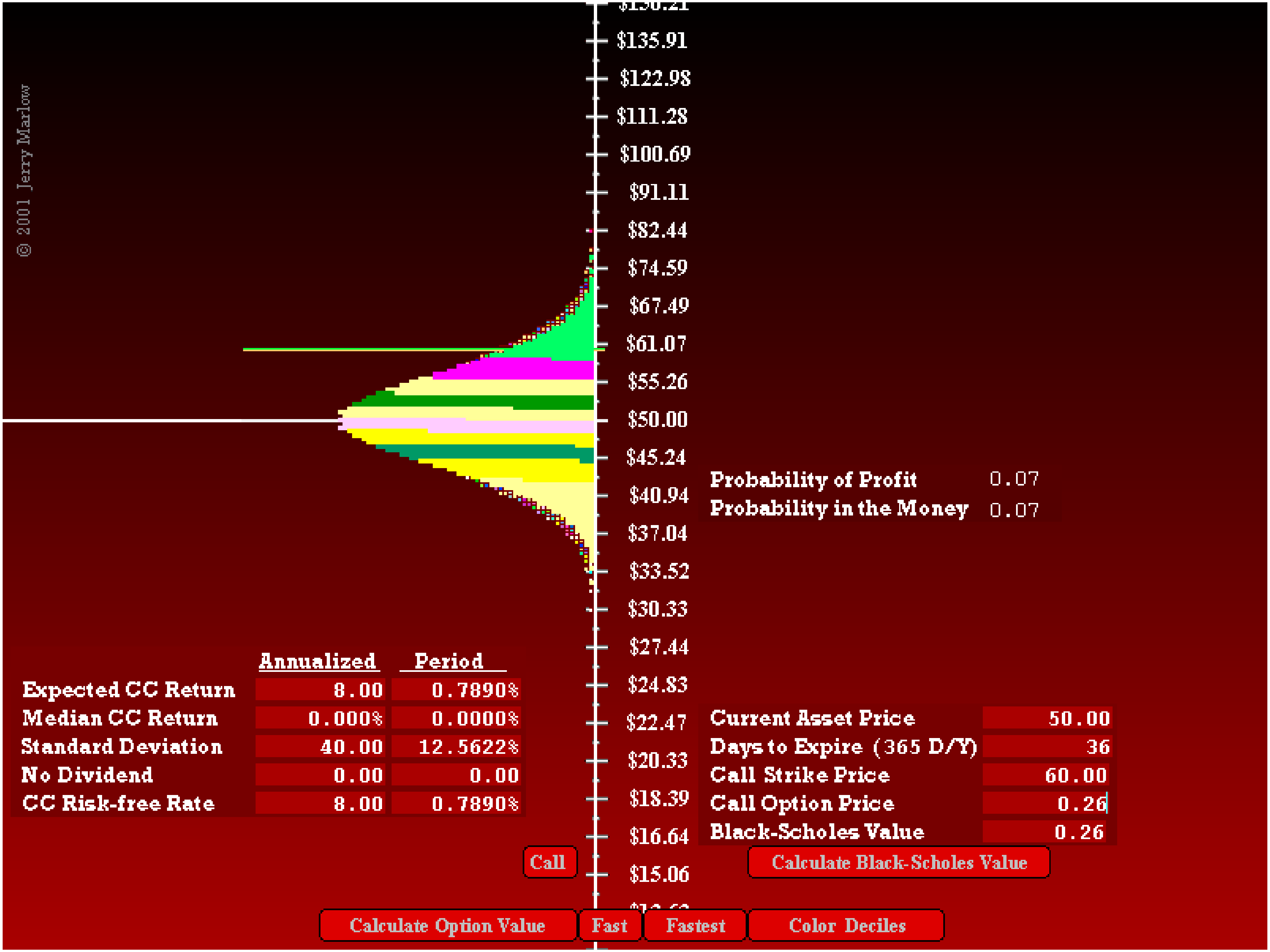
In this case, a ten percent increase in spot price gives a 241% increase in option value.

When the spot price of the underlying increases, the value of a call goes up.

To calculate an approximation of delta, you could follow these steps:

1. Re-establish the baseline call.
2. Increase the spot price by 1%.
3. Find the percentage change in Black-Scholes value.
4. Divide the percentage change in value by 1%.

To get a display of the Black-Scholes value to more decimal places, use the sweep method of calculating its value.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$45.24
$40.94
$37.04
$33.52
$30.33
$27.44
$24.83
$22.47
$20.33
$18.39
$16.64
$15.06
Probability of Profit 0.07
Probability in the Money 0.07
Annualized Period
Expected CC Return 8.00 0.7890%
Median CC Return 0.000% 0.0000%
Standard Deviation 40.00 12.5622%
No Dividend 0.00 0.00
CC Risk-free Rate 8.00 0.7890%
Current Asset Price 50.00
Days to Expire (365 D/Y) 36
Call Strike Price 60.00
Call Option Price 0.26
Black-Scholes Value 0.26
Call
Calculate Black-Scholes Value
Calculate Option Value
Fast
Fastest
Color Deciles

Θ, theta—If the underlying pays no dividends, as a call option's time to expiration grows shorter, fewer little squares are above the strike price. The ones above are not as far above. The option's value goes down.

Θ, theta, is the rate of change of an option's value with changes in its time to maturity.

Let's see how a change in the option's time to maturity changes where the little squares fall relative to the option's strike price. First we redraw our baseline call option.

1. Click three times on Clear.
2. Tab to Expected CC Return. Enter 8.00.
3. For Standard Deviation. Enter 40.00.
4. For Risk-free Rate, enter 8.0.
5. For Current Asset Price, enter 50.00.
6. For Days to Expire, enter 48.
7. For Call Strike Price, enter 60.00
8. Click on Calculate Black-Scholes Value.

The option's Black-Scholes value is $0.46.

9. For Call Option Price, enter 0.46.
10. Click on Color Deciles.

Ten percent of the little squares are above the strike price.

So that we can make a visual comparison, we draw our baseline forecast with little squares.

11. Click on Calculate Option Value Fastest.

Let's see how a shortening of the option's time to maturity changes the value of the call. We'll decrease the time to maturity by twenty-five percent, from 48 days to 36.

12. For Days to Expire, enter 36.
13. Click on Calculate Black-Scholes Value.

The option value is $0.26.

14. For Call Option Price, enter 0.26.
15. Click on Color Deciles.

Volatility, as measured by the standard deviation of the probability distribution, changes with the square root of time. In this example, the period standard deviation decreases from 14.5055% to 12.5622%.

$$(14.5055\%)\left(\sqrt{\frac{36}{48}}\right) = 12.5622\%$$

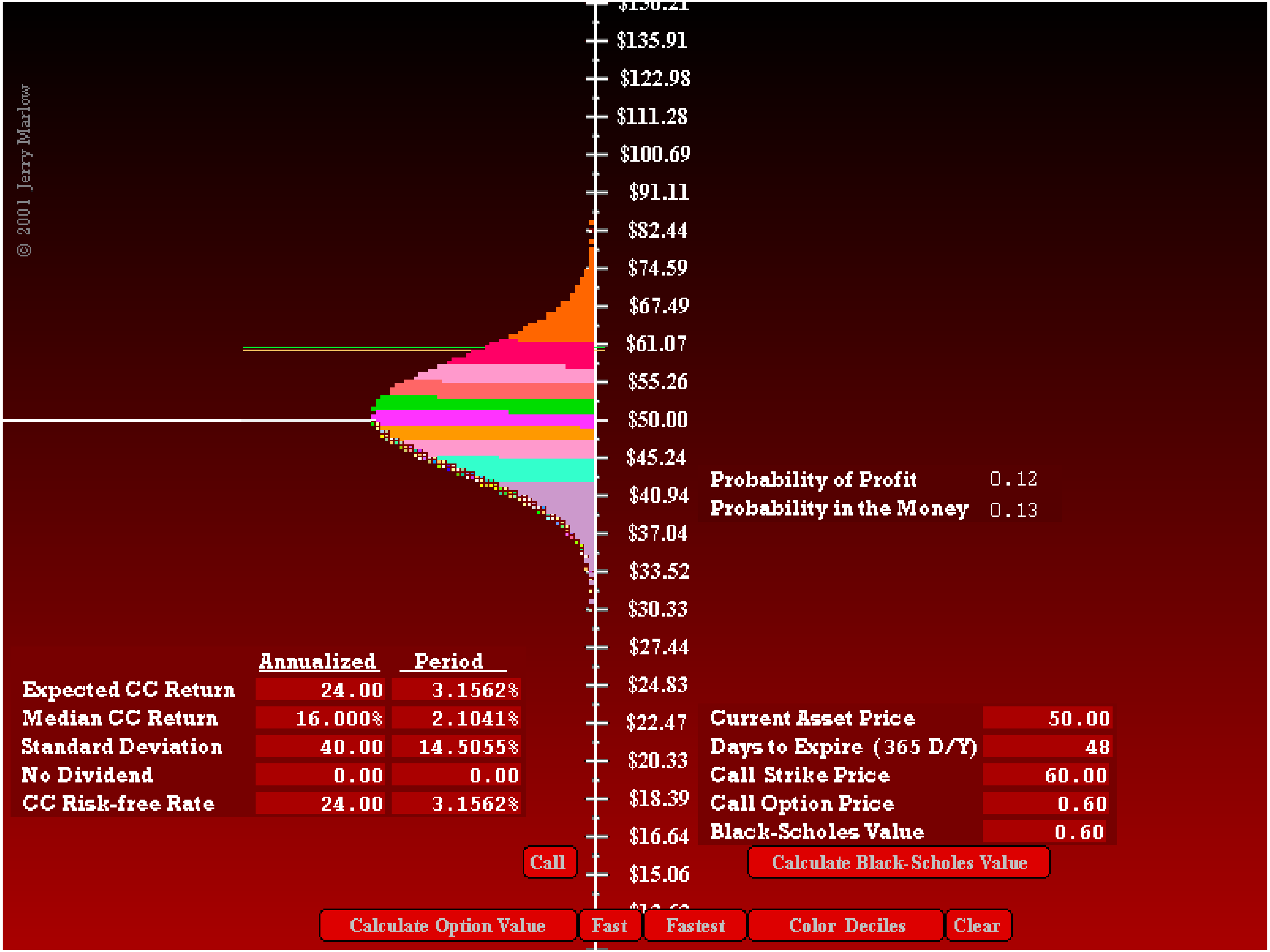
The probability distribution narrows. Fewer little squares are above the strike line. Those that are above don't go up as far. Now seven percent of the little squares are above the strike price. That's a change of (.07 - .10)/.10 = -30%.

The option's value changes by ($0.26 - $0.46)/$0.46 = -43%

When you hold all other factors the same, if the underlying pays no dividends, as a call's time to maturity, shortens, its Black-Scholes value goes down. Sometimes people refer to an option's decline in value with the passage of time as its time decay.

To calculate an approximation of theta, you could follow these steps:

1. Re-establish the baseline call.
2. Decrease the time to expiration by 1 day.
3. Find the percentage change in time to expiration.
4. Find the percentage change in Black-Scholes value.
5. Divide the percentage change in Black-Scholes value by the percentage change in time to expiration.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$45.24
$40.94
$37.04
$33.52
$30.33
$27.44
$24.83
$22.47
$20.33
$18.39
$16.64
$15.06
Probability of Profit 0.12
Probability in the Money 0.13
Annualized Period
Expected CC Return 24.00 3.1562%
Median CC Return 16.000% 2.1041%
Standard Deviation 40.00 14.5055%
No Dividend 0.00 0.00
CC Risk-free Rate 24.00 3.1562%
Current Asset Price 50.00
Days to Expire (365 D/Y) 48
Call Strike Price 60.00
Call Option Price 0.60
Black-Scholes Value 0.60
Call
Calculate Black-Scholes Value
Calculate Option Value
Fast
Fastest
Color Deciles
Clear

Ρ, rho—An increase in the risk-free rate increases the underlying's median return. The number of little squares above the strike price increases. The squares above are higher above. The Black-Scholes value of a call goes up.

Rho is the rate of change of the Black-Scholes value of an option as the risk-free rate changes.

To see the effect, let's redraw our baseline call which has a value of $0.46.

1. Click twice on Clear.
2. Click twice on Hide.
3. Click on Call.
4. Click on Calculate Option Value Fast.
5. Click on Calculate Black-Scholes Value.
6. Tab to Expected CC Return. Enter 8.00.
7. Tab to Standard Deviation. Enter 40.00.
8. For CC Risk-free Rate, enter 8.0.
9. For Current Asset Price, enter 50.00.
10. For Days to Expire, enter 48.
11. For Call Strike Price, enter 60.00.
12. Click on Calculate Black-Scholes Value.

The option's Black-Scholes value is $0.46.

13. For Call Option Price, enter 0.46.
14. Click on Color Deciles.

Ten percent of the little squares are above the strike price.

So that we can make a visual comparison, we draw our baseline forecast with little squares.

15. Click on Calculate Option Value Fastest.

To be able to see the effect of an increase in the risk-free rate on the little squares, we have to change the interest rate a lot. We'll increase it by 200%.

16. For Expected CC Return, enter 24.00.
17. For CC Risk-free Rate, enter 24.00.
18. Click on Calculate Black-Scholes Value.

The option value is $0.60.

19. For Call Option Price, enter 0.60.
20. Click on Color Deciles.

Black-Scholes uses the risk-free rate as the option's expected return. An increase in the risk-free rate increases the underlying's median return. Here it increases it from 0.00% to 16.00%. The increase in the median return lifts the probability distribution.

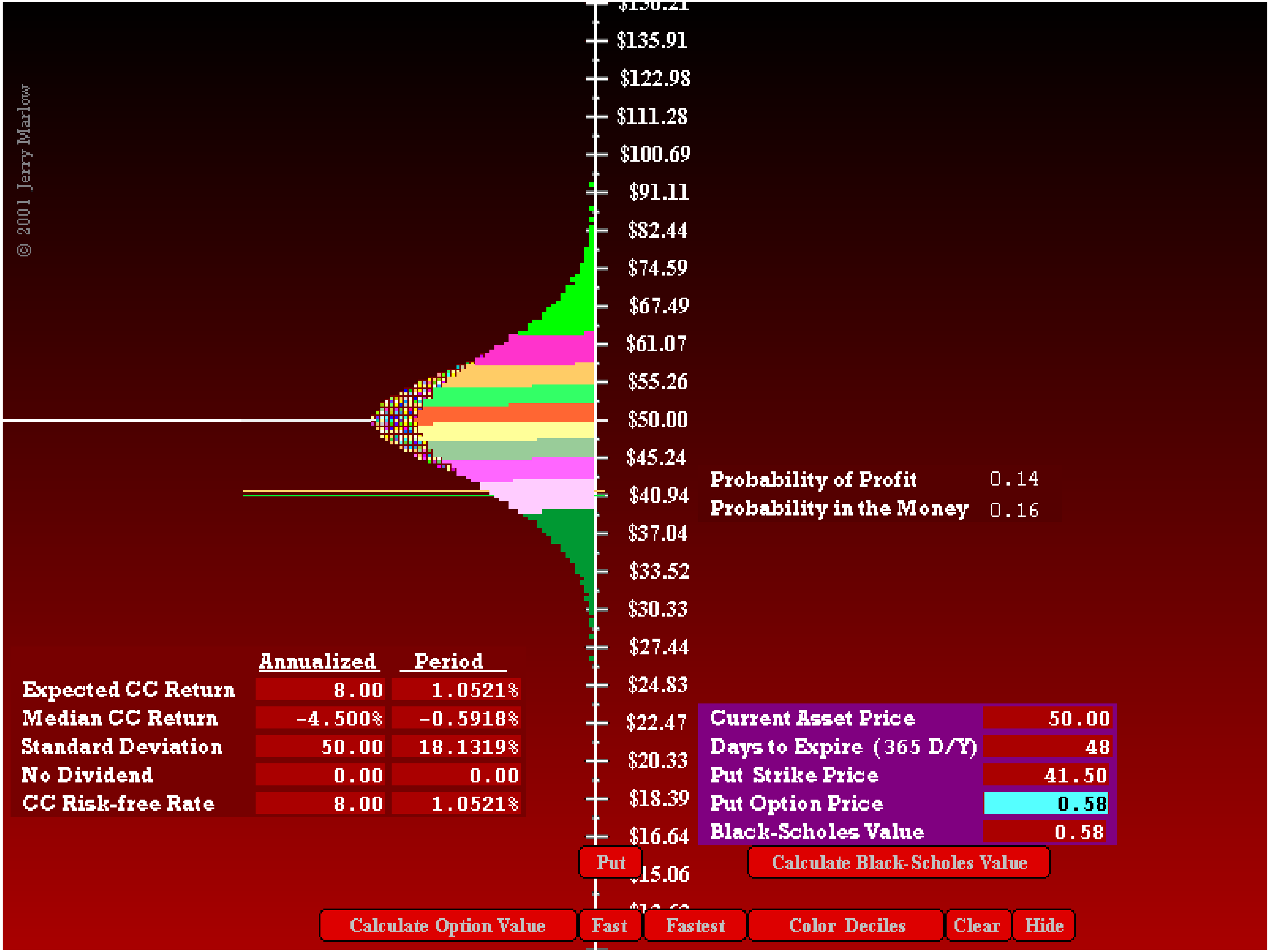
The number of little squares above the strike price increases. Here it goes up by
(0.13 - .10)/.10 = 30%.

The option's value increases. It goes up by
($0.60 - $0.46)/$0.46 = 30%

Holding all other factors the same, under the Black-Scholes assumptions, the higher the risk-free rate, the greater a call's value.

To calculate an approximation of rho, you could:

1. Re-establish the baseline call.
2. Increase the risk-free rate by 1%.
3. Find the percentage change in Black-Scholes value.
4. Divide the percentage change by 1%.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$45.24
$40.94
$37.04
$33.52
$30.33
$27.44
$24.83
$22.47
$20.33
$18.39
$16.64
15.06
Probability of Profit 0.14
Probability in the Money 0.16
Annualized Period
Expected CC Return 8.00 1.0521%
Median CC Return -4.500% -0.5918%
Standard Deviation 50.00 18.1319%
No Dividend 0.00 0.00
CC Risk-free Rate 8.00 1.0521%
Current Asset Price 50.00
Days to Expire (365 D/Y) 48
Put Strike Price 41.50
Put Option Price 0.58
Black-Scholes Value 0.58
Put
Calculate Black-Scholes Value
Calculate Option Value
Fast
Fastest
Color Deciles
Clear
Hide

ς, vega—If volatility increases, the underlying's probability distribution spreads and drops down. The value of a put goes up.

Now we look at the sensitivity of the Black-Scholes value of puts to changes in the underlying's volatility, spot price, the option's time to expiration, and the risk-free rate.

First we establish a baseline put.

1. Click three times on Clear.
2. Click twice on Hide.
3. Click on Put.
4. Click on Calculate Option Value Fastest.
5. Click on Calculate Black-Scholes Value.
6. Expected CC Return, enter 8.00.
7. Tab to Standard Deviation. Enter 40.00.
8. Tab to CC Risk-free Rate. Enter 8.00.
9. For Current Asset Price, enter 50.00.
10. For Days to Expire, enter 48.
11. For Put Strike Price, enter 41.50.
12. Click on Calculate Black-Scholes Value.

The value of the put is $0.26.

13. For Put Option Price, enter 0.26.
14. Click on Color Deciles.

Ten percent of the little squares are below the strike price. Ninety percent are above.

15. Click on Calculate Option Value Fast or Fastest.

Let's see how much the proportion of little squares below the strike price changes when we increase the standard deviation by twenty-five percent.

16. For Standard Deviation, enter 50.00.
17. Click on Calculate Black-Scholes Value.

The option's Black-Scholes value is $0.58.

18. For Put Option Price, enter 0.58.
19. Click on Color Deciles.

Once again, an increase in volatility has lowered the underlying's median return. The probability distribution is both more spread out and a tiny bit lower. The greater spread means that some of the little squares are farther away from the strike price—and, thereby, worth more.

We've gone from ten percent of the little squares below the strike price to sixteen percent. That's a change of (0.16 - .10)/.10 = 60%.

Let's see the percentage change in the option's value :

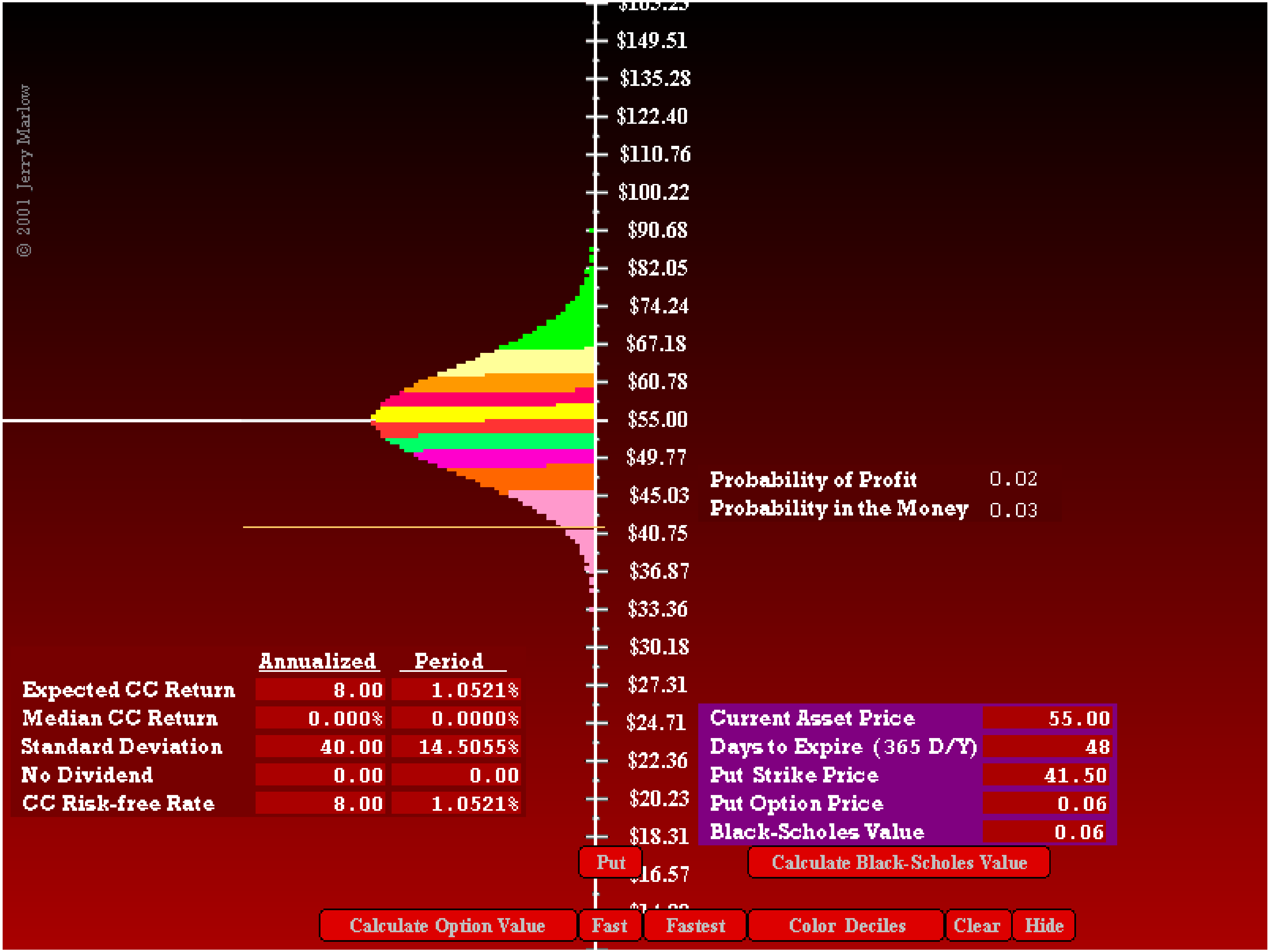
($0.58 - $0.26)/$0.26 = 123%.

When all else stays the same, an increase in volatility increases the value of a put.

To calculate an approximation of vega, you could:

1. Re-establish the baseline put.
2. Increase the standard deviation by 1%.
3. Find the percentage change in Black-Scholes value.
4. Divide the percentage change by 1%.

To get a display of the Black-Scholes value to more decimal places, use the sweep method of calculating its value.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$49.77
$45.03
$40.75
$36.87
$33.36
$30.18
$27.31
$24.71
$22.36
$20.23
$18.31
16.57
Probability of Profit 0.02
Probability in the Money 0.03
Annualized Period
Expected CC Return 8.00 1.0521%
Median CC Return 0.000% 0.0000%
Standard Deviation 40.00 14.5055%
No Dividend 0.00 0.00
CC Risk-free Rate 8.00 1.0521%
Current Asset Price 55.00
Days to Expire (365 D/Y) 48
Put Strike Price 41.50
Put Option Price 0.06
Black-Scholes Value 0.06
Put
Calculate Black-Scholes Value
Calculate Option Value
Fast
Fastest
Color Deciles
Clear
Hide

Δ, delta—When the spot price of the underlying increases, the value of a put goes down

We look at how a change in the underlying's spot price changes the value of a put.

First we re-establish our baseline put.

1. Click twice on Clear.
2. For Expected CC Return, enter 8.00.
3. Tab to Standard Deviation. Enter 40.00.
4. Tab to CC Risk-free Rate. Enter 8.00.
5. Tab to Current Asset Price. Enter 50.00.
6. For Days to Expire, enter 48.
7. For Put Strike Price, enter 41.50.
8. Click on Calculate Black-Scholes Value.

The value of the baseline put is $0.26.

9. For Put Option Price, enter 0.26.
10. Click on Color Deciles.

Ten percent of the little squares are below the strike price.

11. Click on Calculate Option Value Fastest.

Let's see how much the proportion of little squares below the strike price changes when we increase the spot price by ten percent.

12. For Current Asset Price, enter 55.00.
13. Click on Calculate Black-Scholes Value.

The option's Black-Scholes value is $0.06.

14. For Put Option Price, enter 0.06.
15. Click on Color Deciles.

An increase in the spot price of the underlying lifts the probability distribution. It stays the same shape.

Fewer of the little squares are below the strike price. The value of the option goes down.

We've gone from ten percent of the little squares below the strike price to three percent. That's a change of (.03 - .10)/.10 = -70%.

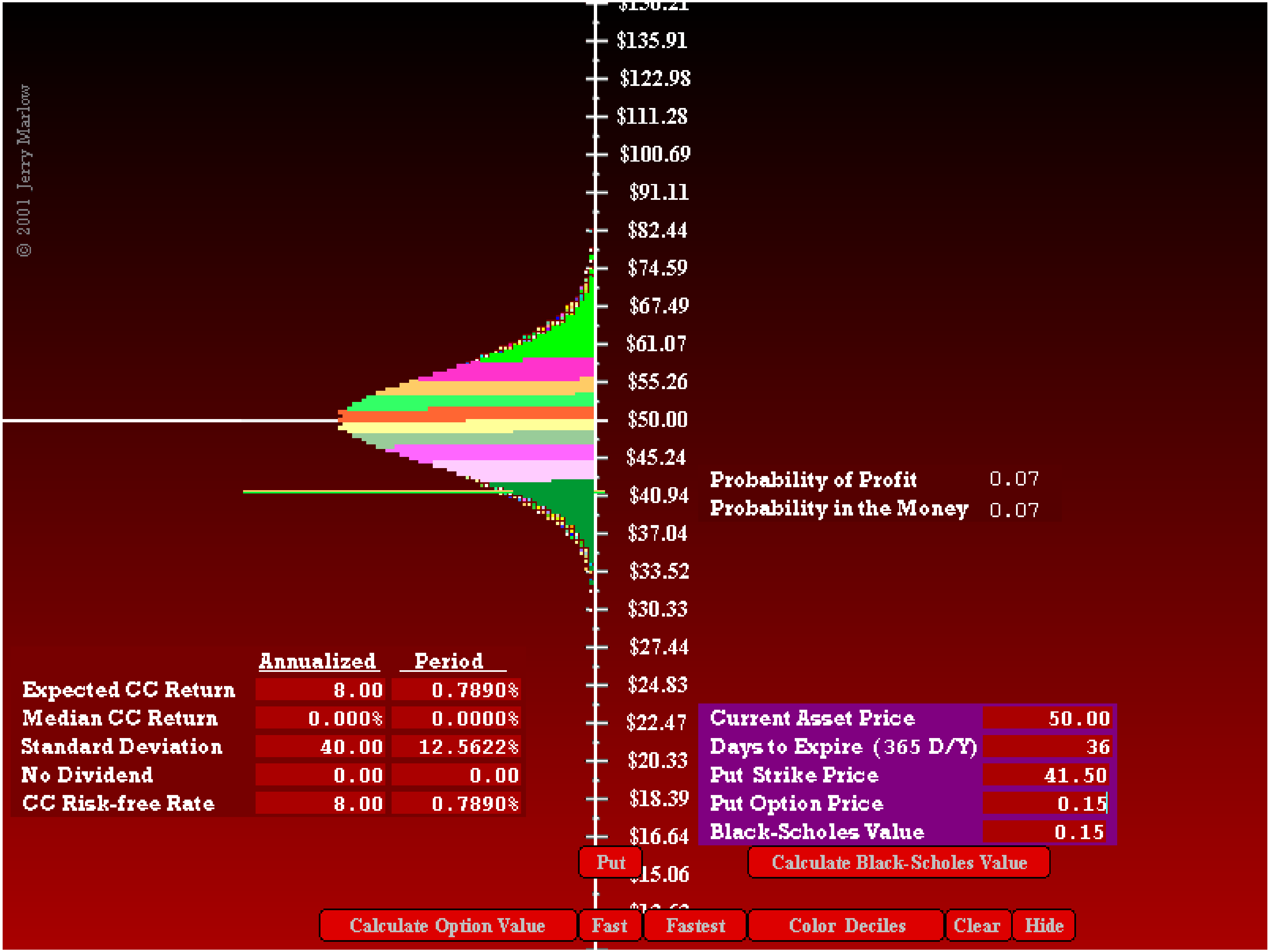
Let's see the percentage change in the option's value :

($0.06 - $0.26)/$0.26 = -77%.

When all else stays the same, an increase in the spot price decrease the value of a put.

To calculate an approximation of delta, you could:

1. Re-establish the baseline put.
2. Increase the spot price by 1%.
3. Find the percentage change in Black-Scholes value.
4. Divide the percentage change by 1%.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$45.24
$40.94
$37.04
$33.52
$30.33
$27.44
$24.83
$22.47
$20.33
$18.39
$16.64
$15.06
Probability of Profit 0.07
Probability in the Money 0.07
Annualized
Period
Expected CC Return 8.00 0.7890%
Median CC Return 0.000% 0.0000%
Standard Deviation 40.00 12.5622%
No Dividend 0.00 0.00
CC Risk-free Rate 8.00 0.7890%
Current Asset Price 50.00
Days to Expire (365 D/Y) 36
Put Strike Price 41.50
Put Option Price 0.15
Black-Scholes Value 0.15
Put
Calculate Black-Scholes Value
Calculate Option Value
Fast
Fastest
Color Deciles
Clear
Hide

Θ, theta—As a put's time to expiration grows shorter, its value goes down. Usually!

Let's look at how a shortening of time to maturity affects a put's Black-Scholes value.

We re-establish our baseline put.

1. Click twice on Clear.
2. For Expected CC Return, enter 8.00.
3. Tab to Standard Deviation. Enter 40.00.
4. Tab to CC Risk-free Rate. Enter 8.00.
5. Tab to Current Asset Price. Enter 50.00.
6. For Days to Expire, enter 48.
7. For Put Strike Price, enter 41.50.
8. Click on Calculate Black-Scholes Value.

The option's value is $0.26.

9. For Put Option Price, enter 0.26.
10. Click on Color Deciles.

Ten percent of the little squares are below the strike price.

11. Click on Calculate Option Value Fastest.

Let's see how a change in the option's time to maturity changes the distribution of little squares. We decrease the time to maturity by twenty-five percent, from 48 days to 36 days.

12. For Days to Expire, enter 36.
13. Click on Calculate Black-Scholes Value.

The option value is $0.15.

14. For Put Option Price, enter 0.15.
15. Click on Color Deciles.

Volatility, as measured by the standard deviation of the probability distribution, changes with the square root of time. In this example, the period standard deviation decreases from 14.5055% to 12.5622%.

$$(14.5055\%)(\sqrt{\frac{36}{48}}) = 12.5622\%$$

The probability distribution narrows.

The median return remains zero. The middle of the probability distribution stays at the same height.

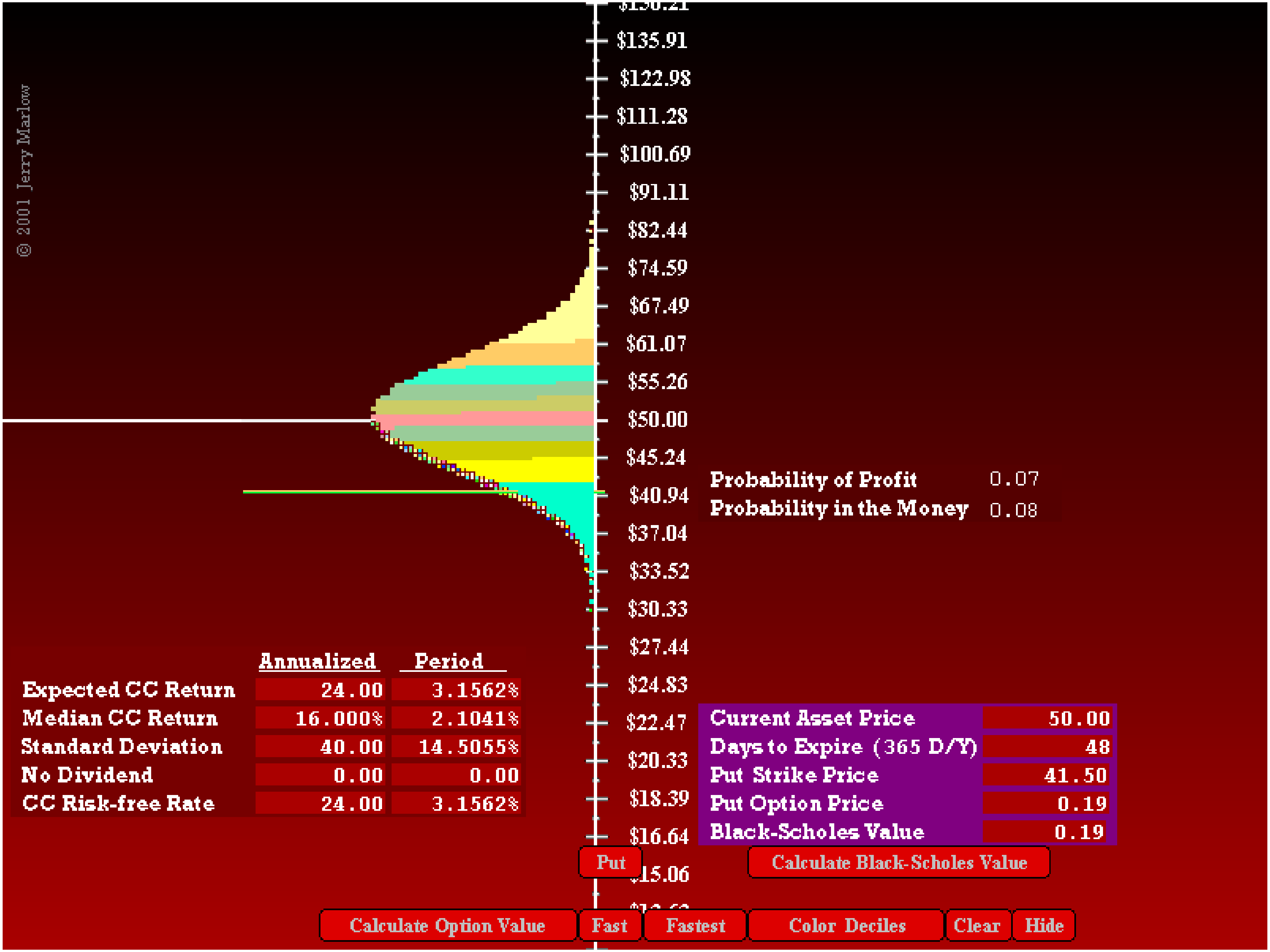
Now seven percent of the little squares are below the strike price. That's a change of
(.07 - .10)/.10 = -30%

The option's value changes by
($0.15 - $0.26)/$0.15 = -42%

Holding all other factors the same, as a European put's time to expiration shortens, its value usually goes down. When we looked at when it is optimal to exercise American puts early, we saw that the put's value does not always go down.

To calculate an approximation of theta, you could:

1. Re-establish the baseline put.
2. Decrease the time to expiration by 1 day.
3. Find the percentage change in time to expiration.
4. Find the percentage change in Black-Scholes value.
5. Divide the percentage change in Black-Scholes value by the percentage change in time to expiration.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$40.94
$37.04
$33.52
$30.33
$27.44
$24.83
$22.47
$20.33
$18.39
$16.64
$15.06
Probability of Profit 0.07
Probability in the Money 0.08
Annualized
Period
Expected CC Return 24.00 3.1562%
Median CC Return 16.000% 2.1041%
Standard Deviation 40.00 14.5055%
No Dividend 0.00 0.00
CC Risk-free Rate 24.00 3.1562%
Current Asset Price 50.00
Days to Expire (365 D/Y) 48
Put Strike Price 41.50
Put Option Price 0.19
Black-Scholes Value 0.19
Put
Calculate Black-Scholes Value
Calculate Option Value
Fast
Fastest
Color Deciles
Clear
Hide

Ρ, rho—An increase in the risk-free rate increases the underlying's median return. The Black-Scholes value of a put goes down.

Let's look at how an increase in the risk-free rate affects a put's Black-Scholes value.

We re-establish our baseline put.

1. Click twice on Clear.
2. For Expected CC Return, enter 8.00.
3. Tab to Standard Deviation. Enter 40.00.
4. Tab to CC Risk-free Rate. Enter 8.00.
5. Tab to Current Asset Price. Enter 50.00.
6. For Days to Expire, enter 48.
7. For Put Strike Price, enter 41.50.
8. Click on Calculate Black-Scholes Value.

The option's value is $0.26.

9. For Put Option Price, enter 0.26.
10. Click on Color Deciles.

Ten percent of the little squares are below the strike price.

11. Click on Calculate Option Value Fastest.

To be able to see the effect on the little squares, we increase the risk-free rate by 200%.

12. Expected CC Return, enter 24.00.
13. For CC Risk-free Rate, enter 24.00.
14. Click on Calculate Black-Scholes Value.

The option value is $0.19.

15. For Put Option Price, enter 0.19.
16. Click on Color Deciles.

Black-Scholes uses the risk-free rate as the option's expected return. An increase in the risk-free rate increases the underlying's median return. The increase in the median return lifts the probability distribution.

The shape of the probability distribution stays the same.

The number of little squares below the strike price decreases. Here it changes by
(.08 - .10)/.10 = -20%.

The option's value goes down. Here it changes by
($0.19 - $0.26)/$0.26 = -27%

Holding all other factors the same, the higher the risk-free rate, the less a put's value.

To calculate an approximation of rho, you could:

5. Re-establish the baseline put.
6. Increase the risk-free rate by 1%.
7. Find the percentage change in Black-Scholes value.
8. Divide the percentage change by 1%.

Using Options to Leverage Your Expected Return

	Annualized	Period
No Dividend	0.00	0.00
CC Risk-free Rate	6.00	3.0082%
Implied Volatility	40.048%	28.3568%

Draw Market-Equilibrium Forecast

Simulate Implied Volatility

Current Asset Price	34.00
Days to Expire (365 D/Y)	183
Call Strike Price	35.00
Call Option Price	3.85

Given a European option's Black-Scholes value or the value of an American call on an underlying that pays no dividends, we can extract the implied volatility of the underlying

Earlier we saw that, given a volatility estimate for an underlying, we can calculate a Black-Scholes value for an option on that underlying.

1. Click three times on Clear.
2. Click twice on Hide.
3. Click on Calculate Black-Scholes Value.
4. For Standard Deviation, enter 40.00.
5. For CC Risk-free Rate, enter 6.0.
6. For Current Asset Price, enter 34.00.
7. For Days to Expire, enter 183.
8. For Call Strike Price, enter 35.00.
9. Click again on Calculate Black-Scholes Value.

The Black-Scholes value for this call is $3.85.

We can go the other way as well. Given a price on a European option, we can extract the volatility estimate that the option price implies.

10. Click on Hide.
11. Click on Calculate Implied Volatility
12. For Call Option Price, enter 3.85.
13. Click again on Calculate Implied Volatility.

The animation extracts from the price an annualized volatility estimate of 40.048%, which is reasonably close to the 40.00% we originally used to calculate the Black-Scholes value. (If you were to calculate the Black-Scholes value for a volatility of 40.048%, you would get $3.85.)

Because European and American calls on underlyings that pay no dividends have the same value, we also can use the Black-Scholes methodology to extract implied volatilities from those American calls.

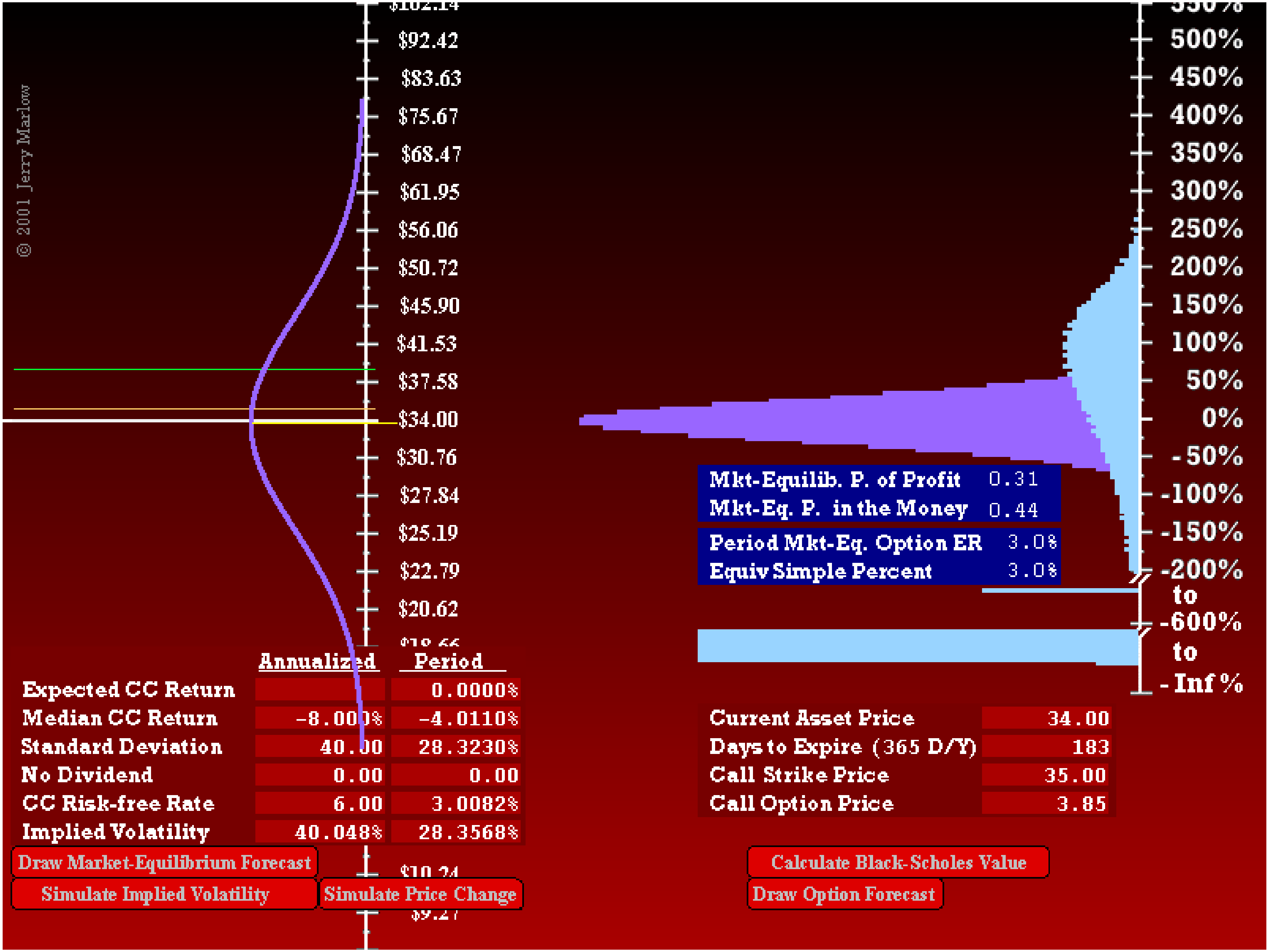
We can think of implied volatility as the market-equilibrium estimate of the uncertainty associated with the underlying's future price movements

As we've seen, the prime determinant of an option price is the uncertainty associated with the underlying's future price movements.

The financial markets are auctions. In stock markets, the expected future value of stocks is auctioned off. In options markets, the expected future volatility of underlyings is auctioned off.

If market participants thought that the volatility estimate implicit in an option's price was out of line, they could perform arbitrage on the option. Arbitrage would force the option price into line with the market's consensus estimate of the underlying's volatility.

Hence, we can think of implied volatility as a market consensus or market-equilibrium view of the uncertainty associated with the underlying's future price movements.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-150%
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to
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to
-Inf%
Mkt-Equilib. P. of Profit 0.31
Mkt-Eq. P. in the Money 0.44
Period Mkt-Eq. Option ER 3.0%
Equiv Simple Percent 3.0%
Annualized
Period
Expected CC Return 0.0000%
Median CC Return -8.000% -4.0110%
Standard Deviation 40.00 28.3230%
No Dividend 0.00 0.00
CC Risk-free Rate 6.00 3.0082%
Implied Volatility 40.048% 28.3568%
Current Asset Price 34.00
Days to Expire (365 D/Y) 183
Call Strike Price 35.00
Call Option Price 3.85
Draw Market-Equilibrium Forecast
Simulate Implied Volatility
Simulate Price Change
Calculate Black-Scholes Value
Draw Option Forecast

If we extract from an option price the implied volatility of a stock and we use the assumption that the expected return of every asset is the risk-free rate, we can draw a risk-neutralized, market-equilibrium forecast for the stock

As you will recall, one of the assumptions of Black-Scholes Options-Pricing Theory is that the investment world is risk neutral. The expected return of every asset is the risk-free rate of return.

We also have seen that we can think of implied volatility as the market-equilibrium estimate of the uncertainty associated with a stock's future price movements.

If we combine the risk-free rate of return with a stock's implied volatility, we have what we might call a risk-neutralized, market-equilibrium forecast for the stock. We can draw this forecast.

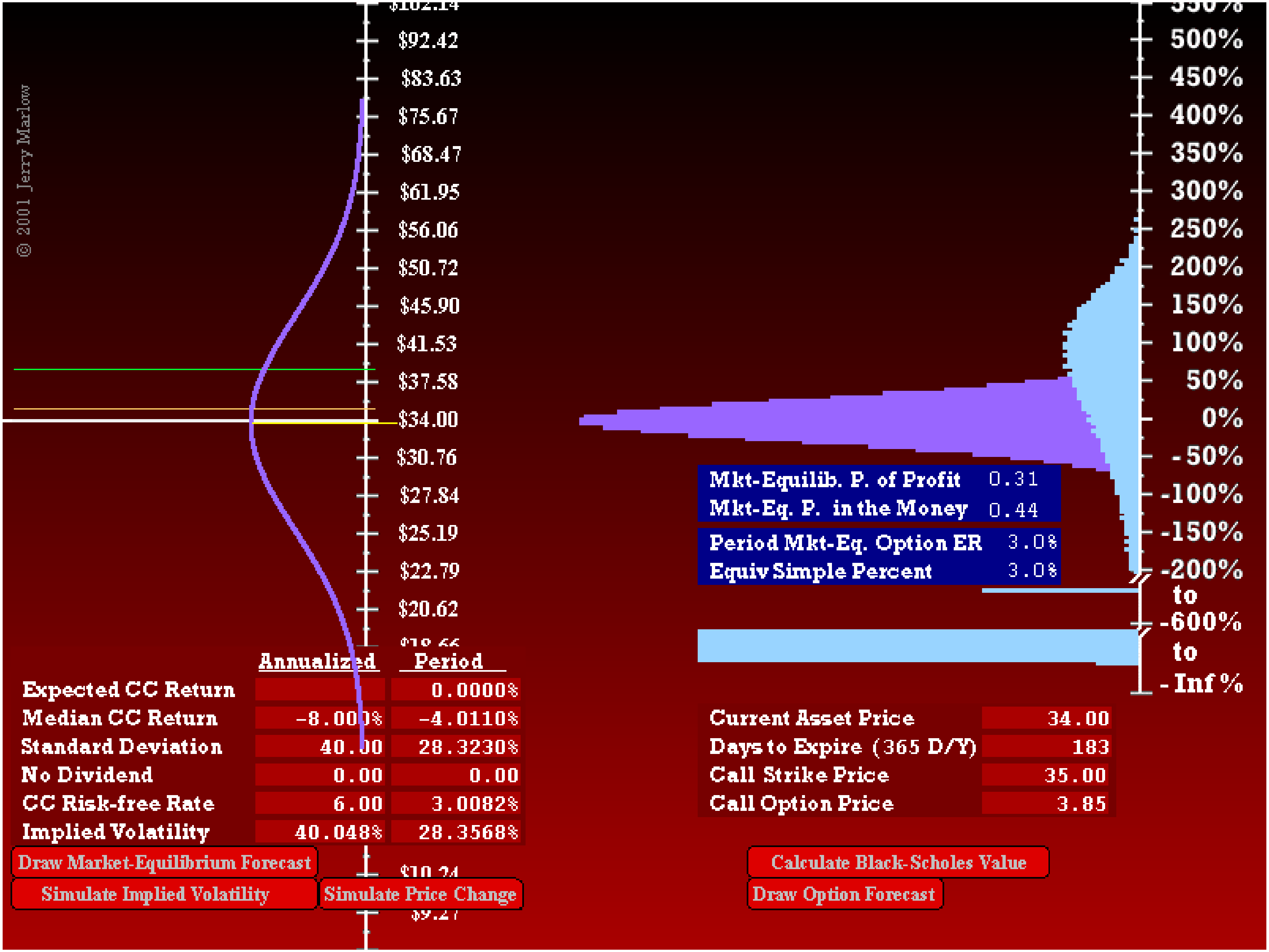
14. Click on Draw Option Forecast.
15. Click on Simulate Price Change.
16. Click on Draw Market-Equilibrium Forecast.

The animation first draws on the option axes the risk-neutralized, implied forecast for the stock.

It then draws the implied forecast for the stock on the price axes.

Finally, the animation draws on the option axes the implied forecast for the option.

For the implied option forecast, the animation calculates the probability of profit, the probability of being in the money, and the period expected return.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Mkt-Equilib. P. of Profit 0.31
Mkt-Eq. P. in the Money 0.44
Period Mkt-Eq. Option ER 3.0%
Equiv Simple Percent 3.0%
Annualized Period
Expected CC Return 0.0000%
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Standard Deviation 40.00 28.3230%
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Days to Expire (365 D/Y) 183
Call Strike Price 35.00
Call Option Price 3.85
Draw Market-Equilibrium Forecast
Simulate Implied Volatility
Simulate Price Change
Calculate Black-Scholes Value
Draw Option Forecast

If you agree with the stock forecast that an option price implies, then the stock and the option have the same expected return

When we calculate the period expected return for the risk-neutralized, market-equilibrium forecast, we get the period risk-free rate of return—in this case 3.0%. The period expected return for the option is the same as the period expected return for the stock.

This is as it should be. Remember, one of the Black-Scholes assumptions is that all assets have the same expected return—the risk-free rate.

The two probability distributions drawn on the options axes are very different. Nevertheless their average returns are the same.

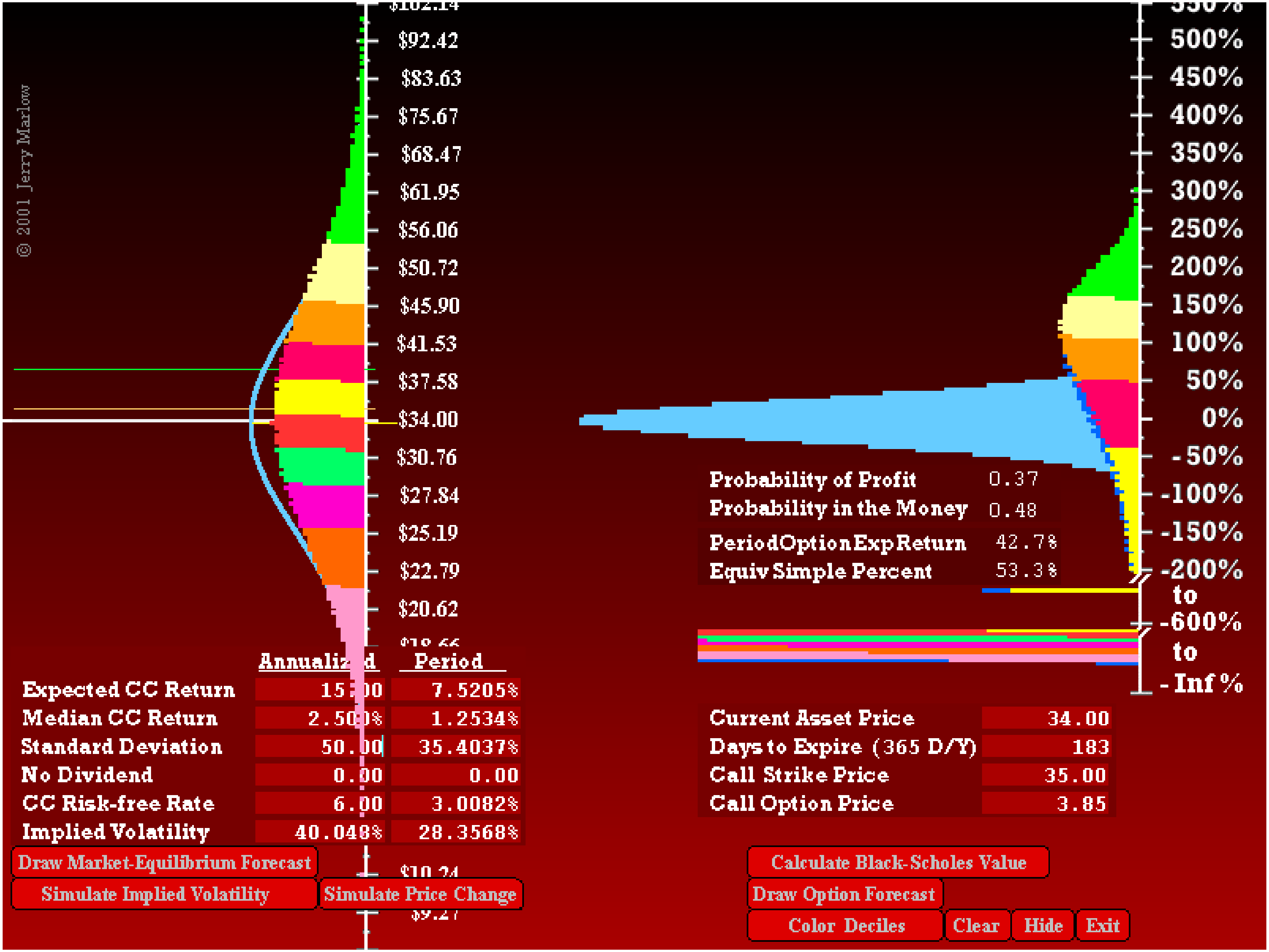
If you agreed with this forecast and you invested in the stock, you would expect your return to be somewhere between roughly 85% and –85%.

If you agreed with this forecast and you invested in the option, you would expect your return to be somewhere between 250% and negative infinity.

Whether you invested in the stock or in the option, your expected return would be 3.0%.

Investing in the option and investing in the stock are equally fair bets. The average outcomes are the same.

If you agreed with the implied forecast, you would have little or no incentive to invest in the option instead of in the stock. You would be taking on a greater exposure to uncertainty with no increase in your expected return.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$27.84
$25.19
$22.79
$20.62
Annualized
Period
Expected CC Return 15.00 7.5205%
Median CC Return 2.500% 1.2534%
Standard Deviation 50.00 35.4037%
No Dividend 0.00 0.00
CC Risk-free Rate 6.00 3.0082%
Implied Volatility 40.048% 28.3568%
Draw Market-Equilibrium Forecast
Simulate Implied Volatility
Simulate Price Change
500%
450%
400%
350%
300%
250%
200%
150%
100%
50%
0%
-50%
-100%
-150%
-200%
to
-600%
to
-Inf%
Probability of Profit 0.37
Probability in the Money 0.48
PeriodOptionExpReturn 42.7%
Equiv Simple Percent 53.3%
Current Asset Price 34.00
Days to Expire (365 D/Y) 183
Call Strike Price 35.00
Call Option Price 3.85
Calculate Black-Scholes Value
Draw Option Forecast
Color Deciles
Clear
Hide
Exit

If you disagree with the stock forecast that an option price implies, then you can use the option to leverage your expected return

Let's say you do not agree with the risk-neutralized, market-equilibrium forecast. You think that, instead of having an expected return of 6.0% and a standard deviation of 40.048%, this stock has an annualized expected return of 15% and a standard deviation of 50%.

Given your forecast, will the option also have an expected return of 15%?

Let's see.

17. For Expected CC Return, enter 15.00.
18. For Standard Deviation, enter 50.00.
19. Click on Color Deciles.

The animation draws your forecast for the stock and your forecast for the option.

The difference in the stock forecasts is not enormous. However, given your forecast for the stock, the period expected return for the option is not 15%, but 42.7%.

The option's structure leverages the difference between the implied forecast and your forecast.

When you disagree with the risk-neutralized, market-equilibrium forecast, you can use options to leverage your expected return.

Black-Scholes Assumptions (Part III)

Nasdaq-100® Index Options **NDX**
Current Price: 1731.16 **European** **Mar 19**
Annualized Nasdaq-100® Index Dividend Yield: 0.11%

Calls	Bid	Ask	Mid
Jun 1100. (NDV FB-E)	643.1	665.1	654.1
Jun 1400. (NDV FH-E)	396.0	418.0	407
Jun 1600. (NDV FL-E)	262.2	284.2	273.2
Jun 1700. (NDV FN-E)	206.9	228.9	217.9
Jun 1800. (NDV FP-E)	161.8	179.8	170.8
Jun 1900. (NDV FR-E)	122.6	140.6	131.6
Jun 2050. (NDX FA-E)	80.1	92.1	86.1
Jun 2100. (NDX FB-E)	68.1	80.1	74.1
Jun 2200. (NDX FD-E)	48.3	60.3	54.3
Jun 2300. (NDX FF-E)	36.3	42.3	39.3
Jun 2400. (NDX FH-E)	25.2	31.2	28.2

Calls	Bid	Ask	Mid
Sep 1100. (NDV IB-E)	679.6	701.6	690.6
Sep 1400. (NDV IH-E)	456.7	478.7	467.7
Sep 1600. (NDV IL-E)	335.3	357.3	346.3
Sep 1700. (NDV IN-E)	283.4	305.4	294.4
Sep 1800. (NDV IP-E)	237.4	259.4	248.4
Sep 1900. (NDV IR-E)	199.1	217.1	208.1
Sep 2050. (NDX IA-E)	148.3	166.3	157.3
Sep 2100. (NDX IB-E)	133.7	151.7	142.7
Sep 2200. (NDX ID-E)	107.9	125.9	116.9
Sep 2300. (NDX IF-E)	86.1	104.1	95.1
Sep 2400. (NDX IH-E)	70.9	82.9	76.9

Does the behavior of the financial markets conform to the Black-Scholes assumptions?

Thus far, blithely we have been assuming that the behavior of the financial markets conforms to the Black-Scholes assumptions.

But does it?

We've already noted that the Black-Scholes model is designed to value and analyze European options. While many index options are European, most stock options are American.

Before you rely too heavily on the Black-Scholes methodologies, you may want to consider how well the behavior of the financial markets conforms to each of the assumptions.

We now look at some of the discrepancies between the Black-Scholes assumptions and market behavior.

As you will recall, one of the assumptions is:

- Traders and investors incur no transaction costs such as bid-ask spreads or brokerage fees and commissions.

In reality, traders and investors face bid-ask spreads and incur transactions costs.

For a given option, the implied volatility of the bid price differs from the implied volatility of the ask price.

Using data from the table above, we can extract and compare implied volatilities for bid and ask prices. We'll use the data for the **Nasdaq-100® Index Options** call option Jun 1600 which has a bid price of 262.2 and an ask price of 284.2. This is a European option. The index pays a dividend yield of 0.11%. Such a tiny dividend yield has no practical impact. Even so, we shall include it in our calculations.

1. Click three times on Clear.
2. Click twice on Hide.
3. Click on Simulate Price Change.
4. Click on Draw Your Forecast.
5. Click once on Hide.
6. Click on Call.
7. Click on Calculate Implied Volatility.
8. For CC Risk-free Rate, enter 6.0.
9. For Current Asset Price, enter 1731.16.
10. For Days to Expire, enter 74.
11. For Call Strike Price, enter 1600.
12. For Call Option Price, enter the bid price of 262.2.
13. Click on Enter Div Yield.
14. For Continuous Div Yield, enter 0.11.
15. Click on Calculate Implied Volatility.

Using the bid price as the Black-Scholes value gives us an annualized implied volatility of 59.770%.

16. Click on Draw Market-Equilibrium Forecast.

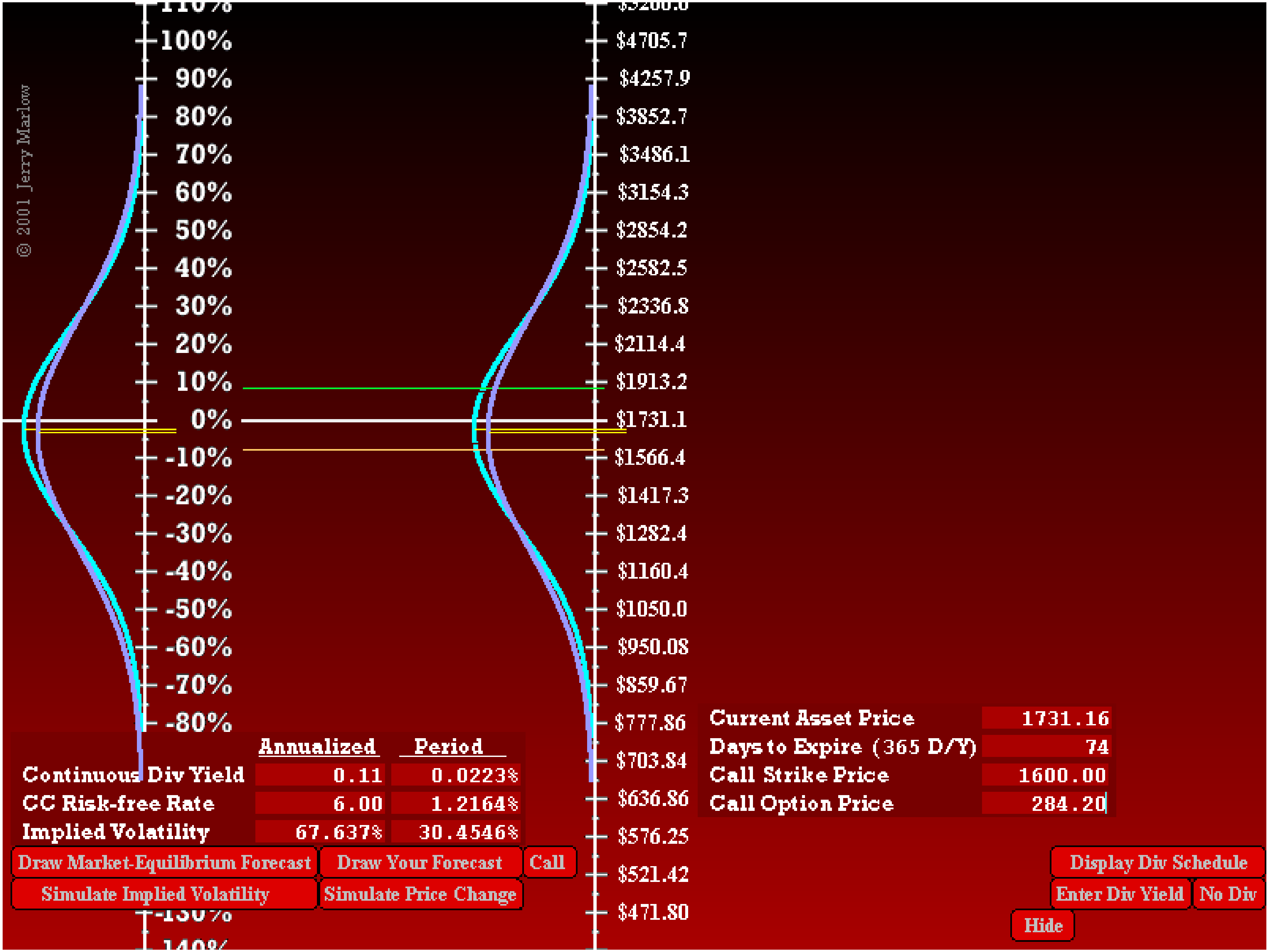
The animation draws the period forecast implicit in the bid price.

17. For Call Option Price, enter the ask price of 284.2.
18. Click on Calculate Implied Volatility.

Using the ask price as the Black-Scholes value gives us an implied volatility of 67.637%.

19. Click on Draw Market-Equilibrium Forecast.

The animation draws the forecast implicit in the ask price.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$1566.4
$1417.3
$1282.4
$1160.4
$1050.0
$950.08
$859.67
$777.86
$703.84
$636.86
$576.25
$521.42
$471.80
Annualized
Period
Continuous Div Yield 0.11 0.0223%
CC Risk-free Rate 6.00 1.2164%
Implied Volatility 67.637% 30.4546%
Current Asset Price 1731.16
Days to Expire (365 D/Y) 74
Call Strike Price 1600.00
Call Option Price 284.20
Draw Market-Equilibrium Forecast
Draw Your Forecast
Call
Simulate Implied Volatility
Simulate Price Change
Display Div Schedule
Enter Div Yield
No Div
Hide

The bid price and the ask price give us different implied volatilities. To get an advantage from buying an option, you have to beat the forecast implicit in the ask price.

An option's bid and ask prices give us different implied volatilities. To calculate an underlying's implied volatility, which price do you use?

It depends on your purpose. If you want to see the risk-neutralized, market-equilibrium forecast implicit in an option price, you may want to use the midpoint of the bid and ask prices.

If you want to consider whether to buy an option because you disagree with the risk-neutralized, market-equilibrium forecast, you may want to look at the forecast implicit in the ask price. To get an advantage from buying the option, this is the forecast you have to beat.

Nasdaq-100® Index Options **NDX**
Current Price: 1731.16 **European** **Mar 19**
Annualized Nasdaq-100® Index Dividend Yield: 0.11%

Calls	Bid	Ask	Mid
Jun 1100. (NDV FB-E)	643.1	665.1	654.1
Jun 1400. (NDV FH-E)	396.0	418.0	407
Jun 1600. (NDV FL-E)	262.2	284.2	273.2
Jun 1700. (NDV FN-E)	206.9	228.9	217.9
Jun 1800. (NDV FP-E)	161.8	179.8	170.8
Jun 1900. (NDV FR-E)	122.6	140.6	131.6
Jun 2050. (NDX FA-E)	80.1	92.1	86.1
Jun 2100. (NDX FB-E)	68.1	80.1	74.1
Jun 2200. (NDX FD-E)	48.3	60.3	54.3
Jun 2300. (NDX FF-E)	36.3	42.3	39.3
Jun 2400. (NDX FH-E)	25.2	31.2	28.2
Jun 2500. (NDX FJ-E)	18.6	21.6	20.1
Jun 2600. (NDX FL-E)	12.8	15.8	14.3
Jun 2700. (NDX FN-E)	9.0	10.5	9.8
Jun 2800. (NDX FP-E)	6.3	7.8	7.1
Jun 2900. (NDX FR-E)	4.7	5.7	5.2
Jun 3050. (NDY FA-E)	2.9	3.9	3.4
Jun 3100. (NDY FB-E)	2.5	3.5	3.0
Jun 3200. (NDY FD-E)	2.0	2.7	2.4
Jun 3300. (NDY FF-E)	1.5	2.3	1.9
Jun 3400. (NDY FH-E)	1.0	1.7	1.4
Jun 3500. (NDY FJ-E)	0.5	1.2	0.9

Calls	Bid	Ask	Mid
Sep 1100. (NDV IB-E)	679.6	701.6	690.6
Sep 1400. (NDV IH-E)	456.7	478.7	467.7
Sep 1600. (NDV IL-E)	335.3	357.3	346.3
Sep 1700. (NDV IN-E)	283.4	305.4	294.4
Sep 1800. (NDV IP-E)	237.4	259.4	248.4
Sep 1900. (NDV IR-E)	199.1	217.1	208.1
Sep 2050. (NDX IA-E)	148.3	166.3	157.3
Sep 2100. (NDX IB-E)	133.7	151.7	142.7
Sep 2200. (NDX ID-E)	107.9	125.9	116.9
Sep 2300. (NDX IF-E)	86.1	104.1	95.1
Sep 2400. (NDX IH-E)	70.9	82.9	76.9

Do option prices imply that a stock's volatility will be constant over a given investment horizon?

Black-Scholes Options-Pricing Theory also makes the following assumption:

- The volatility of a stock's price path is constant over the investment horizon.

If the market's behavior conformed to this assumption, then, to calculate option prices for a given stock at different strike prices, an options trader would always enter into the Black-Scholes formula the same volatility estimate.

We would come along to extract the implied volatility from the option prices. From different option prices at different strike prices, we would always get the same implied volatility.

In reality, do we get the same implied volatilities?

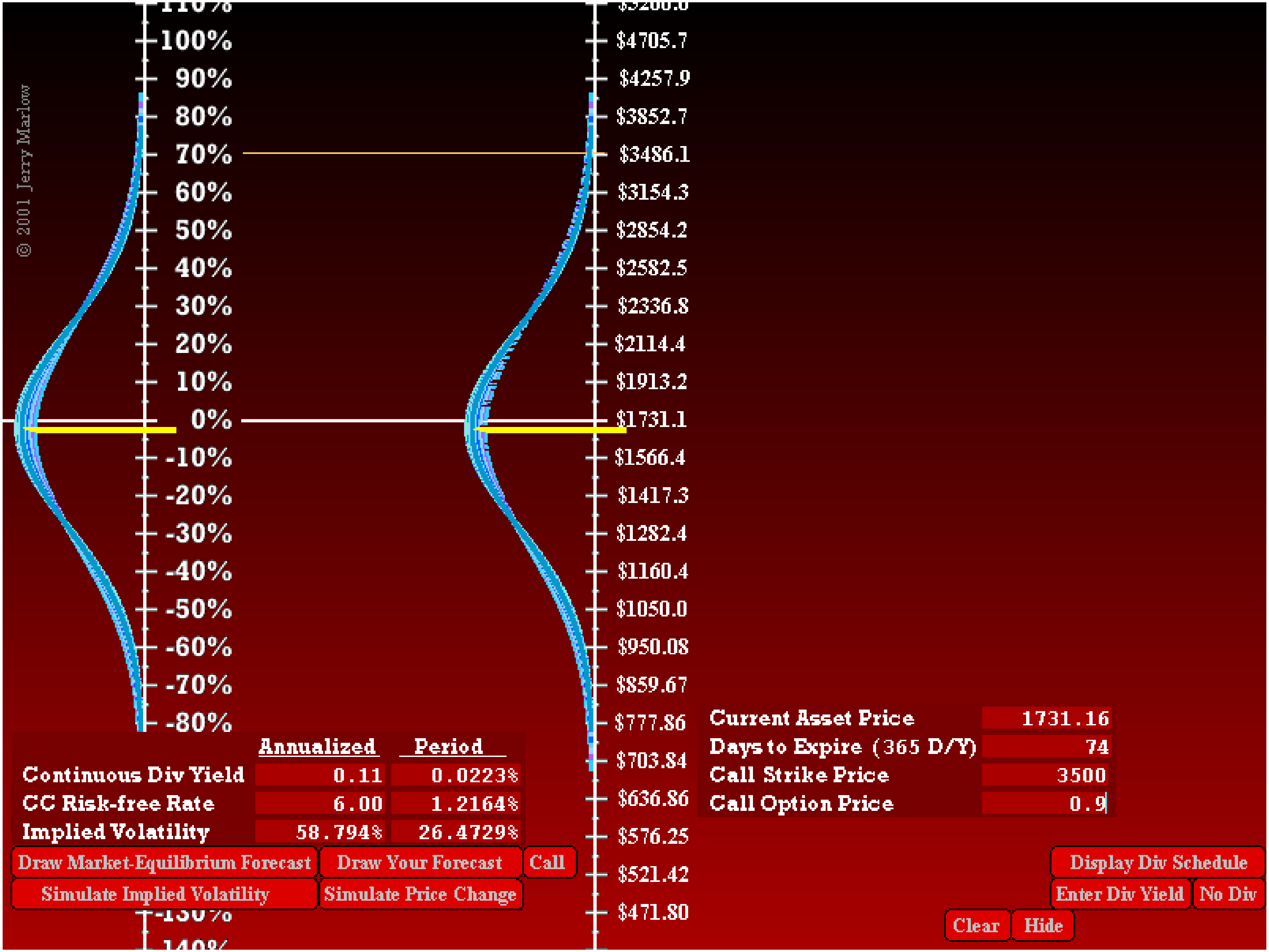
No. It doesn't matter whether we use the bid prices, the ask prices or the midpoints. We get different implied volatilities at different strike prices.

Continuing to use data from the table above, we extract implied volatilities for options at different strike prices. We'll use the June 1 call data.

20. Click once on Clear.

For each option:

A. Enter the Call Strike Price.
B. For Call Option Price, enter the midpoint Price.
C. Click on Calculate Implied Volatility.
D. Click on Draw Market-Equilibrium Forecast.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$1050.0
$950.08
$859.67
$777.86
$703.84
$636.86
$576.25
$521.42
$471.80
Annualized
Period
Continuous Div Yield 0.11 0.0223%
CC Risk-free Rate 6.00 1.2164%
Implied Volatility 58.794% 26.4729%
Current Asset Price 1731.16
Days to Expire (365 D/Y) 74
Call Strike Price 3500
Call Option Price 0.9
Draw Market-Equilibrium Forecast
Draw Your Forecast
Call
Simulate Implied Volatility
Simulate Price Change
Display Div Schedule
Enter Div Yield
No Div
Clear
Hide

Different strike prices give us different implied volatilities for the same underlying. This is the so-called "volatility smile."

What we get is a range of implied volatilities. The implied volatility is generally the lowest for strike prices near the spot or current market price of the underlying. (Though that's not happening here with the volatile March 19 Nasdaq 100 Index.)

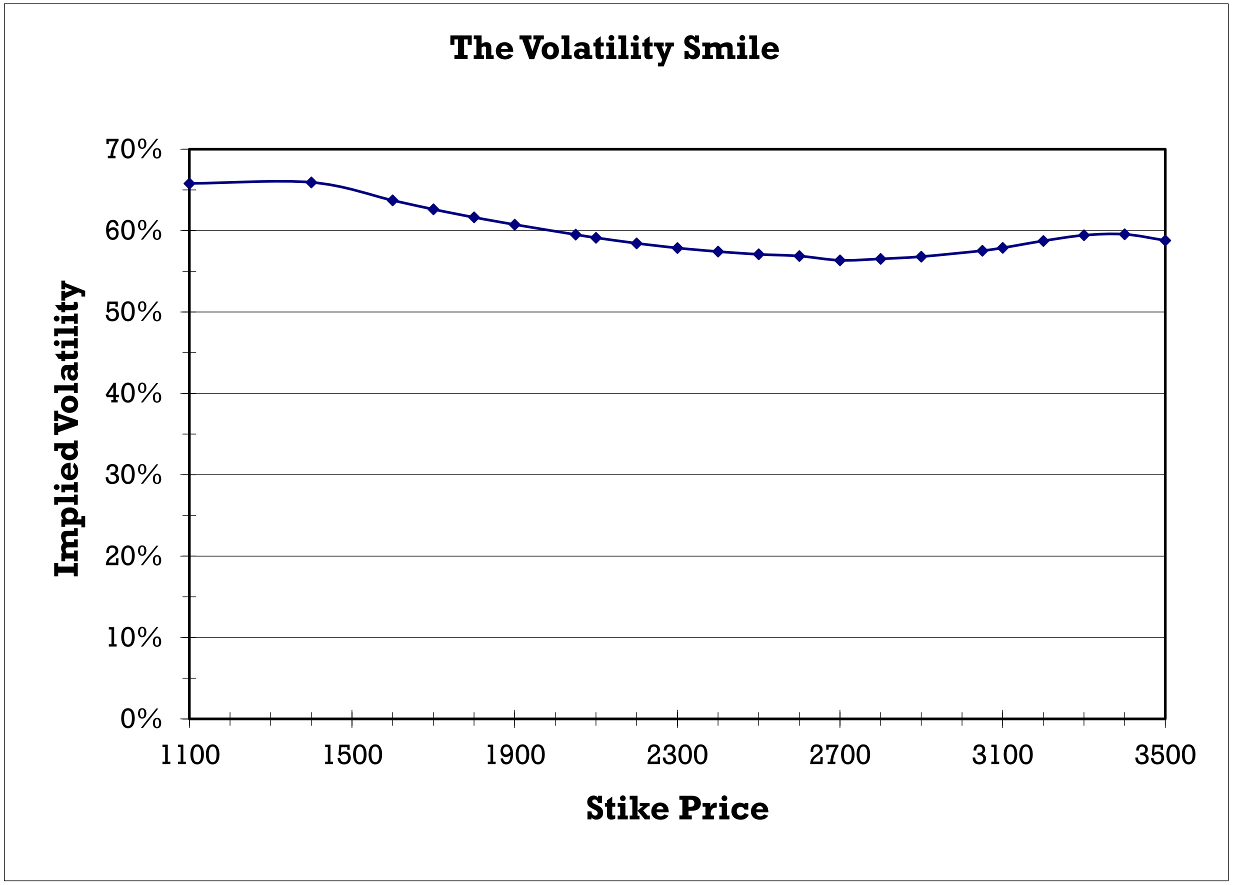
For strike prices above and below the spot price, the implied volatilities generally go up. When you graph implied volatility against strike price, the graph looks like a smile—hence the volatility smile. (Maybe a crooked smirk here?)

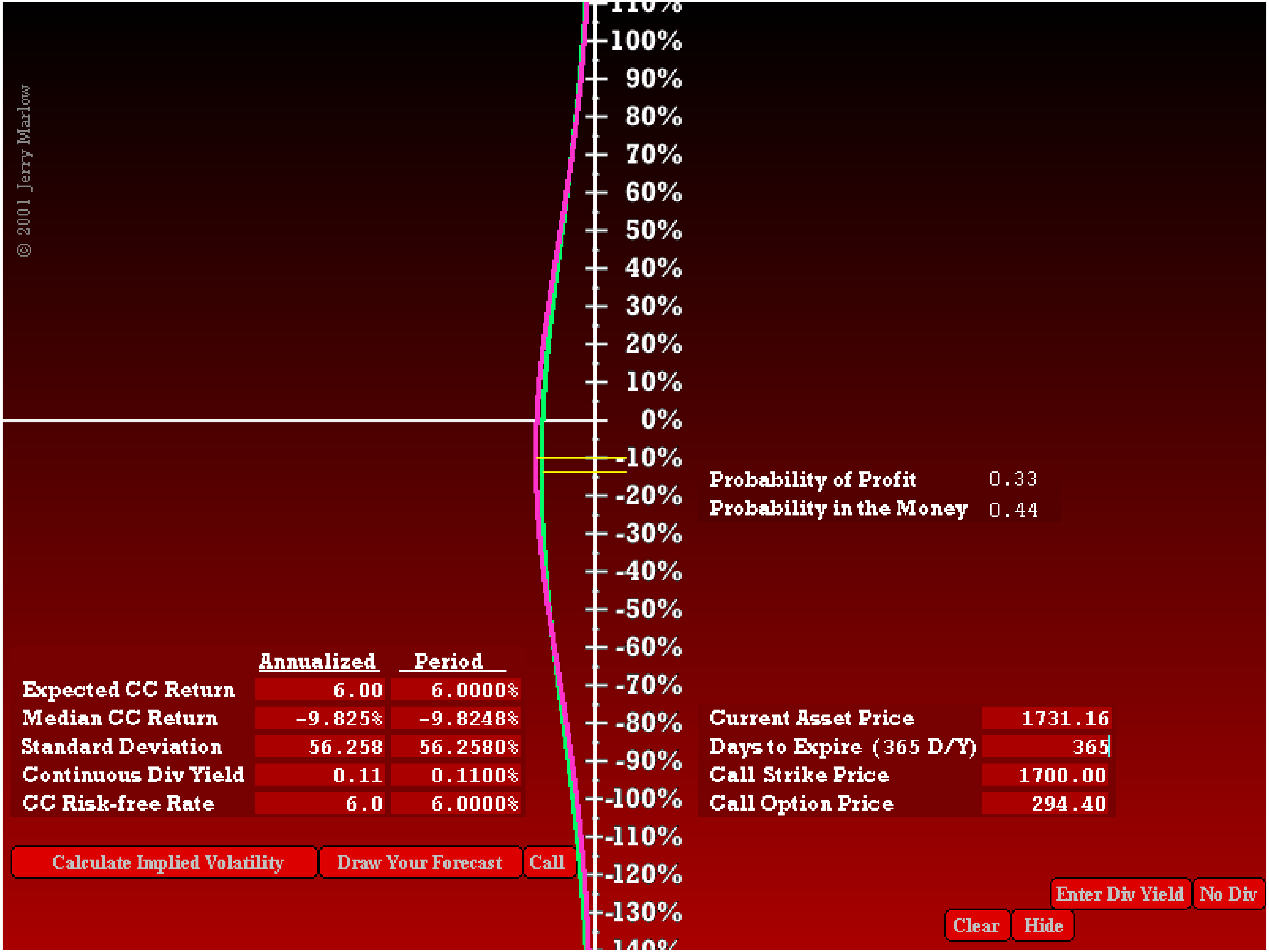
Why does the volatility smile exist?

The most common explanation is that market participants do not agree with the Black-Scholes assumption of constant volatility. Instead they believe that, if the spot price changes much, the option's volatility will go up. The option will become more valuable.

Some market gurus put forth the idea that market participants who fear a crash bid up the price of deep out-of-the-money puts. Those relatively higher prices translate into higher implied volatilities for those options.

Given the smile, when we use an option price to calculate a stock's implied volatility, the volatility estimate applies only for that particular strike price and time to expiration. It does not hold for different strike prices or for different expiration dates.

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-70%
-80%
-90%
-100%
-110%
-120%
-130%
Probability of Profit 0.33
Probability in the Money 0.44
Annualized
Period
Expected CC Return 6.00 6.0000%
Median CC Return -9.825% -9.8248%
Standard Deviation 56.258 56.2580%
Continuous Div Yield 0.11 0.1100%
CC Risk-free Rate 6.0 6.0000%
Current Asset Price 1731.16
Days to Expire (365 D/Y) 365
Call Strike Price 1700.00
Call Option Price 294.40
Calculate Implied Volatility
Draw Your Forecast
Call
Enter Div Yield
No Div
Clear
Hide

For a given strike price, different expiration dates give us different implied volatilities. This is the term structure of volatility.

It may be tempting to extract an implied volatility from an option and then extend or contract that volatility to a different investment horizon. This technique doesn't work.

Options with different expiration dates usually give us different annualized implied volatilities. This difference is referred to as the term structure of volatility.

To analyze the term structure, practitioners use the implied volatilities for at-the-money strike prices.

1. Click three times on Clear.
2. Click twice on Hide.
3. Click on Call.
4. For Current Asset Price, enter 1731.16.
5. For Days to Expire, enter 74.
6. For Call Strike Price, enter 1700.00.
7. For Call Option Price, enter 217.90.
8. Click on Calculate Implied Volatility.
9. Click on Enter Div Yield.
10. For Continuous Div Yield, enter 0.11.
11. For CC Risk-free Rate, enter 6.0.
12. Click on Calculate Implied Volatility.

For the June call, the annualized implied volatility is 62.617%.

13. Click on Draw Your Forecast.
14. For Expected CC Return, enter 6.0.
15. For Standard Deviation, enter 62.617.
16. For Days to Expire, enter 365.
17. Click on Draw Your Forecast.

The animation draws the annualized implied volatility for the option that expires in 74 days.

18. Tab to Days to Expire. Enter 166.
19. For Call Option Price, enter 294.4.
20. Click on Calculate Implied Volatility.

For the September call at the same strike price, the annualized implied volatility is 56.258%.

21. For Standard Deviation, enter 56.258.
22. For Days to Expire, enter 365.
23. Click on Draw Your Forecast.

The animation draws the annualized implied volatility of the option that expires in 166 days.

Be sensitive to the ways in which the financial markets may not conform to the Black-Scholes assumptions

If you go back and review the other Black-Scholes assumptions, you'll be aware that the financial markets may not conform to a number of them:

- Stock returns expressed as geometric rates of return may not be perfectly normally distributed.
- Price changes may not be perfectly lognormally distributed.
- Geometric Brownian motion may not perfectly characterize potential price paths.
- The volatility of a stock's price path may not be constant over the investment horizon.
- Stock-price paths may not be continuous. Prices may jump.
- Traders and investors may not be able to trade continuously.
- The financial markets may not be perfectly liquid.
- Traders and/or investors may not be able to borrow at the risk-free rate.
- The risk-free rate may change over a given investment horizon.
- Traders and investors incur transaction costs.
- Arbitrage opportunities may crop up.
- Many investors probably are not risk neutral.

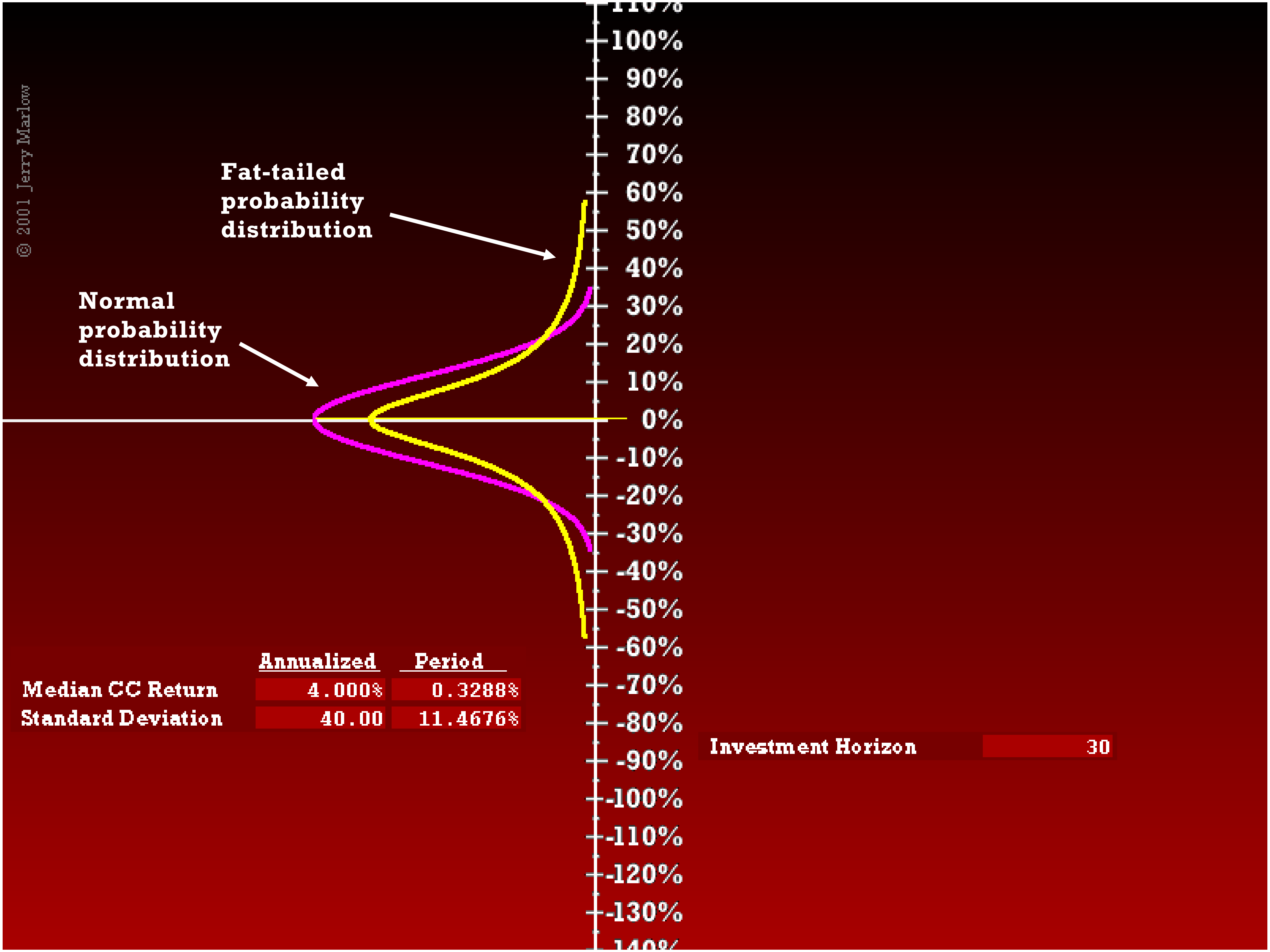
Given the fact that the financial markets do not conform to many of the Black-Scholes assumptions, you might expect traders and analysts to abandon the Black-Scholes methodologies altogether. However, they have not done so. They have developed additional methodologies, but Black-Scholes is still very much with us.

The Black-Scholes way of thinking gives you tools with which to analyze relationships among forecasts, hedging costs, probability distributions, expected return, and potential price paths.

The Black-Scholes model usually is the take-off point for developing other option-pricing models. Understanding Black-Scholes makes it easier to understand alternative pricing models.

In many respects, the markets behave in ways that are close enough to the Black-Scholes assumptions for the methodologies to prove useful. The central premises of the theory hold true. Volatility drives the cost of hedging options positions. The cost of hedging drives options prices.

When you use the Black-Scholes model, be sensitive to the ways in which financial markets may not conform to underlying assumptions.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Fat-tailed probability distribution
Normal probability distribution
110%
100%
90%
80%
70%
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0%
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-70%
-80%
-90%
-100%
-110%
-120%
-130%
-140%
Annualized
Period
Median CC Return
4.000%
0.3288%
Standard Deviation
40.00
11.4676%
Investment Horizon
30

Theoreticians keep building models they hope will model the financial markets more accurately than does Black-Scholes

Many theoreticians and practitioners object to a number of the Black-Scholes assumptions. Accordingly, they keep building models that seek to remedy their objections.

The Black-Scholes model prices easily only European options and American calls on underlyings that pay no dividends. To price other American options, binomial models are much easier to use than Black's approximation. Binomial models look automatically at the potential value of early exercise.

Black-Scholes assumes that stock returns expressed as geometric rates of return are normally distributed. Many theoreticians argue that the actual distribution of geometric rates of return are more fat tailed than in a normal distribution. In their models, they may use more fat-tailed distributions. The graphs above show the difference between a normal distribution and one type of more fat-tailed distribution.

Black-Scholes assumes that the potential price paths of a stock can be characterized by a geometric-Brownian-motion model. This approach assumes that, at any given instant, the probabilities are equal for an up or down percentage change about a stock's average return. Some alternative models assign or allow the user to assign different probabilities to up ticks and down ticks.

Black-Scholes assumes that the volatility of a stock's price path is constant over the investment horizon. Alternative models may adjust expectations about future volatility to the outcomes of price-path simulations. If a volatile potential price path increases a model's expectations about future volatility, the pricing result may be similar to using a fat-tailed distribution with a constant-volatility assumption.

Black-Scholes assumes that the risk-free rate of interest will not change over the investment horizon. Alternative models may incorporate interest rates that vary in keeping with the yield curve.

Theoreticians generally assume that the market prices of options are the correct prices. That is, they assume market prices accurately embody consensus expectations about future stock prices, interest rates, volatility, and the nature of a stock's potential price paths.

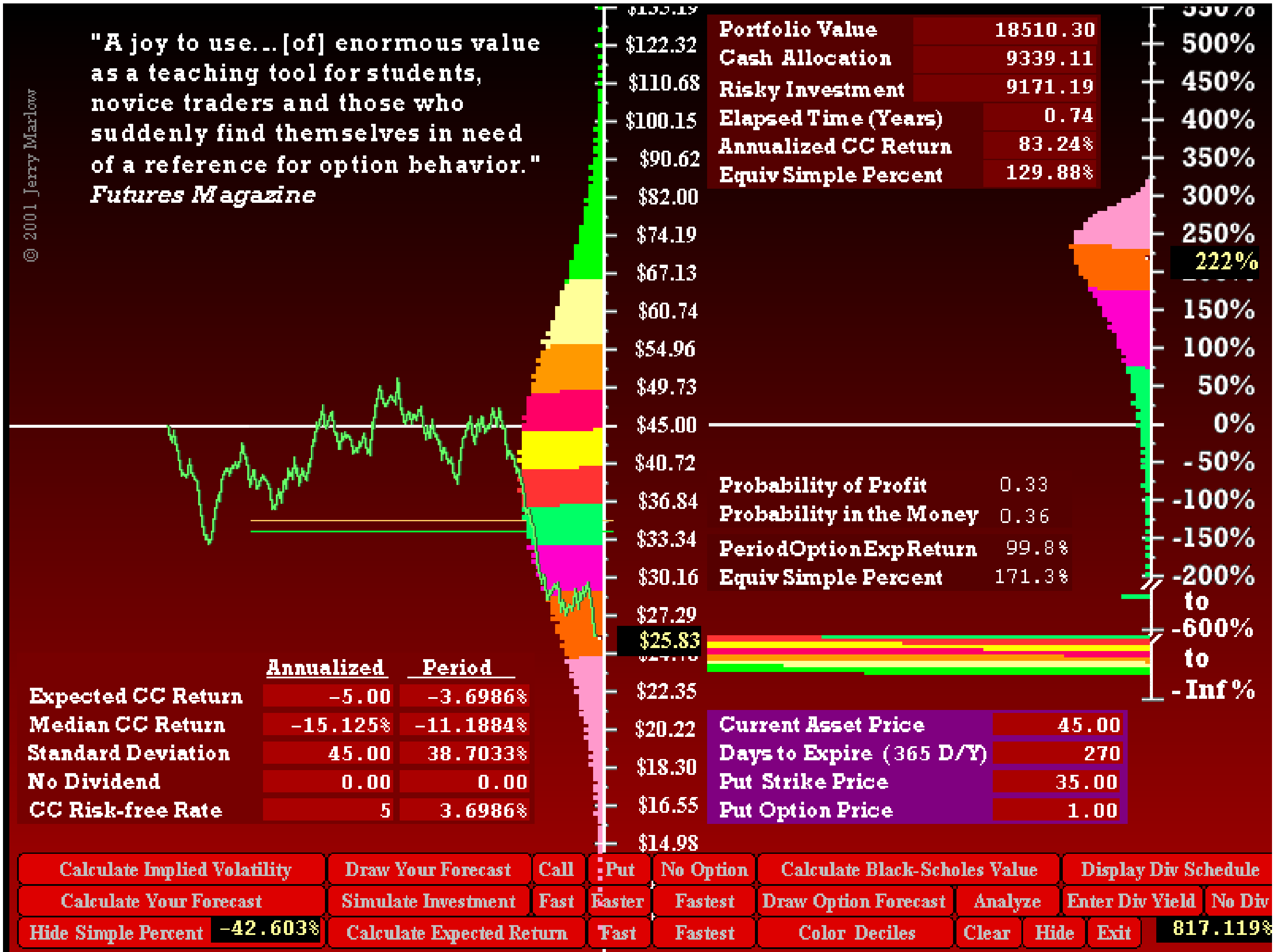
In this view, the goal of an option-pricing model is, from a given set of inputs, to produce option prices that are consistent with market prices across all strike prices and expirations. Going in the other direction, from all the options on a given stock, the model would extract the same implied volatility. Such a model would make the Black-Scholes volatility smile go away.

When you began your study of Black-Scholes, you may have thought the model complex and difficult. Compared to the alternatives, Black-Scholes is simple, elegant, and easy to use. If you have a good understanding of how option values change as all those little squares move around relative to the strike price, you'll be able to understand what the theoreticians and self-appointed experts are arguing about. You'll even be able to join in the fray.

As *Black-Scholes Made Easy* goes to press, the author has begun to experiment with animations that would allow an investor to draw probability distributions of whatever shape he or she believes best describes the market behavior of underlyings. If you are interested in keeping abreast of those experiments, on the *Black-Scholes Made Easy CD* double-click on StayInTouch.htm and send the author an e-mail.

(The animation is not programmed to draw the illustration above.)

"A joy to use... [of] enormous value as a teaching tool for students, novice traders and those who suddenly find themselves in need of a reference for option behavior."
Futures Magazine



"The animation is a good one. A deep insight into the price process pops out clearly. In my next semester course, *Black-Scholes Made Easy* will be a useful tool for giving students the necessary intuitive view of the matter."
Professor Lucio Geronazzo
Mathematical Models for Financial Markets
Università degli Studi di Firenze

"Your animation is simply the best. It makes every part of Black-Scholes so understandable. I knew Black-Scholes option pricing model was important and use it every day in my trading, but never knew why or how it worked. Your book and CD-ROM provided me a clear understanding of the model, and best of all, all I need to understand the model was basic mathematical background and some common sense. Thank you very much."
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Equity & Derivatives Team
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